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ORIGINAL PAPER

Analyses of fold profiles using cubic Bézier curve

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Abstract

Matching curves to represent diverse fold morphologies is an active research field in structural geology that can have far-reaching implication in resource studies and in engineering geology. Further, some of the recent literatures show that 2D and 3D restoration and best fit of folds have been researched actively in petroleum geosciences. The significant advantage of the method presented here using cubic Bézier curve is that the profile of a fold could be represented in terms of *four* "controlling points", which can be re-synthesized in 2D graphical plot using the spreadsheet programme such as Microsoft Excel, Apache Open Office Spreadsheet etc. by simply developing a tabular spreadsheet based on the equation of cubic Bézier curve. The method is simple and has been tested successfully to synthesize a few fold profiles by changing the values of coordinates of the controlling points and on photographs of two natural examples of folds. Bézier curves of different order have been used along with multi-paradigm programming/numerical computing software such as MATLAB, mathematical symbolic computation program: Wolfram Mathematica, and vector graphics designing. However, it is not easy for all learners or researchers to rapidly use or develop such programmes. On the other hand, spreadsheets programmes, both commercial and open-source, well known to the majority of the population having a general knowledge in computers.

Keywords Fold morphology · Curve fitting · Bézier curve · Structural geology

Introduction

Since fold morphologies can indicate rock rheology (Fletcher 1979) and bulk shortening (Ghassemi et al. 2010), categorization and analyses of fold shapes constitute important exercises in structural geology (e.g., Hjlle et al. 2013; partial review in Mukherjee 2014; also see Gogoi and Mukherjee 2019). Morphologic representation of structures in terms of ideal curves and graphic tools has gained popularity in petroleum geology, engineering geology and in modeling complicated structures in geology (e.g., Zhong et al. 2004; de Cemp 1999). Some of the techniques involve harmonic coefficient of the Fourier series (Stabler 1968), power functions (Bastida et al. 1999), quadratic Bézier curves through 'Bézier Fold Profiler' (e.g., Liu et al. 2009a, b), static Hamilton–Jacobi equation (Hjlle et al. 2013), NURB curve (Gogoi et al. 2017; Biswas and Mukherjee submitted), cubic Bezier curves, conic sections, power functions and super-ellipses in FOLD PROFILER, MATLAB (Lisle et al. 2006 and references therein) etc. Bézier curves (Bézier 1966, 1967) are widely used in Computer-Aided Design (CAD) (De Paor 1996). Following this trend, the present work utilizes cubic Bézier (Bernstein-Bézier) equations to simulate a range of fold profiles. Masood and Ejaz (2010) have already elaborated use of such curves in a wide range of graphic applications. Cubic Bézier curve more appropriately can approximate shapes in 2D than quadratic Bézier curves (Gueziec 1996; Chun et al. 2009). Venkataraman (2009) presents standard properties of the cubic Bézier equations/curves. An alternative technique has been presented in this work to synthesize fold profiles using spreadsheet programmes.

To construct the Bézier curve (Bézier 1966, 1967), this work first time utilizes a spreadsheet in Microsoft Excel, which allows tabular mathematical calculations and plotting

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of data in different forms of graphs. With slight adjustments of coordinates of the ‘controlling points’ of the cubic Bézier curves plotted in the Spreadsheet, best-fit curve could be traced over a photograph of the fold and imported directly into the Graph Plot Area. This curve could be exported in raster format or could be recorded by noting the coordinates obtained after fitting, which could later be used for re-synthesis of the same fold profile.

The fundamental equation of Bézier curve is (Biswas and Lovell 2008; Vince 2010; Janke 2015; Sederberg 2016):

$$C(t) = \sum_{i=0}^{n} P_i B_{i,n}(t), \quad t \in [0, 1]. \tag{1}$$

Here, value of parameter (variable) t varies between 0 and 1. P_i is the location of controlling or handling points, n is the degree of curve. A set of $(n+1)$ numbers of control points $P_0, P_1, \ldots, P_n$ exist for the curve. $B_{i,n}$ is called the blending functions, and $B_{i,n}(t)$ is called the Bernstein polynomials function, which can be represented as (Agoston 2005; Vince 2010):

$$Bi, n(t) = \binom{n}{i}(1-t)^{n-i}t^i, \quad i = 0, 1, \ldots, n. \tag{2}$$

Here, $\binom{n}{i}$: *binomial coefficient*, i.e. $\frac{n!}{i!(n-i)!}$. If $n=2$, $B_{0,2}=(1-t)^2$, $B_{1,2}=2t\,(1-t)$ and $B_{2,2}=t^2$. Substituting the values of $B_{0,2}$, $B_{1,2}$ and $B_{2,2}$, Eq. (1) gives a quadratic Bézier equation of second degree. Liu et al. (2009) (*a,b*) used quadratic Bézier curves in their work to simulate folds:

$$C(t) = (1-t)^2P_0 + 2(1-t)(t)P_1 + t^2P_2 \quad t \in [0, 1]. \tag{3}$$

Here, the points P_0, P_1 and P_2: controlling points. The curve is tangent to $P_1 - P_0$ and $P_n - P_{n-1}$ at the end points.

A cubic Bézier curve (Fig. 1) may be derived from Eq. (1) (Marsh 2005; Agoston 2005; Lisle et al. 2006; Biswas and Lovell 2008):

$$C(t) = (1-t)^3P_0 + 3(1-t)^2(t)P_1 + 3(1-t)t^2P_2 + t^3P_3 \quad t \in [0, 1]. \tag{4}$$

Coordinates correspond to controlling points P_0, P_1, P_2 and P_3. Here, P_0 and P_3 are (x_i, y_i): x_0, y_0, x_1, y_1, x_2, y_2 and x_3, y_3, respectively. P_0 and P_3 are called the beginning- and the ending points, and here the other two (P_1 and P_2) are intermediate points that control the shape of the curve (Parent 2012).

Equation (4) can be split into two equations substituting coordinates of controlling points (Davies et al. 1986; Vince 2010):

$$X(t) = (1-t)^3x_0 + 3(1-t)^2(t)x_1 + 3(1-t)t^2x_2 + t^3x_3, \tag{5}$$

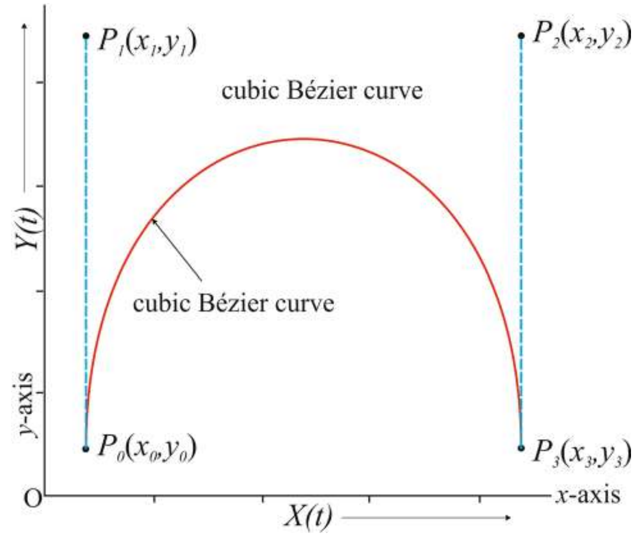

Fig. 1 An example of a cubic Bézier curve produced from plotting of $X(t)$ and $Y(t)$ (based on Eqs. 5 and 6) for $t \in [0, 1]$ is an the control points P_0, P_1, P_2 and P_3 with coordinates (x_0, y_0), (x_1, y_1), (x_2, y_2) and (x_3, y_3), respectively

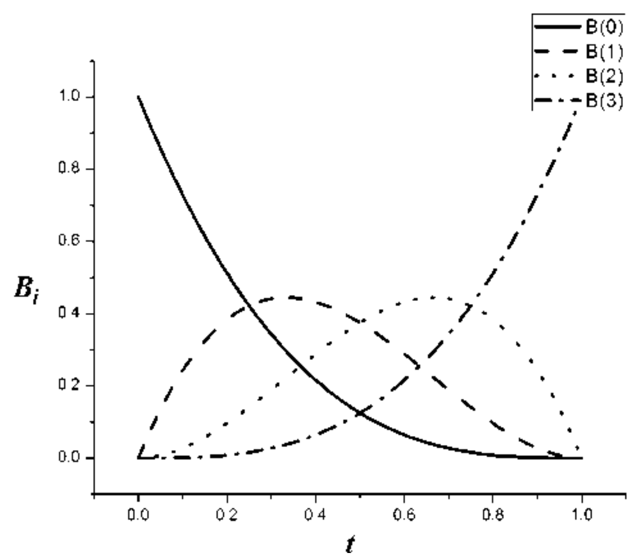

Fig. 2 Variation of $B_{i,n}(t)$ for $t \in [0,1]$ (value of t varies from 0 to 1 as shown in Fig. 3)

$$Y(t) = (1-t)^3y_0 + 3(1-t)^2(t)y_1 + 3(1-t)t^2y_2 + t^3y_3. \tag{6}$$

Figure 1 represents a Bézier Curve produced from Eqs. (5) and (6) with controlling points P_o (x_0, y_0), P_1 (x_1, y_1), P_2 (x_2, y_2) and P_3 (x_3, y_3). The properties of Bernstein polynomials (Eq. 2) are fixed for a specified degree (2nd, 3rd, 4th etc.) of a Bézier Curve. In our case, we deal with 3rd degree (cubic) Bézier equation (Eq. 4). Variation of Bernstein polynomials in Eq. (4) can be plotted as shown in Fig. 2 for $t \in [0, 1]$.

Therefore, the shape of the curve can be altered with the help of P_0 (x_0, y_0), P_1 (x_1, y_1), P_2 (x_2, y_2) and P_3 (x_3, y_30), which are the controlling points of the curve as shown in Fig. 1.

Methods

Spreadsheet programmes are more common and easy to use as programming/coding is not required for simple arithmetical calculations and for plotting graphs. Therefore, different types of folds are simulated using a cubic Bézier equation (Eqs. 5 and 6) with the help of Microsoft Excel spreadsheet (Repository files). The layout and functions used in the spreadsheet have been shown in Fig. 3. The parametric variable 't' controls the shape of the curve and its value has been increased from 0.00 to 1.00. To plot the curve, a scattered smooth line graph has been selected and columns X(t) and Y(t) act as a data source for the graph. The smoothness of the curve depends on the degree of fractions used for variable 't'. We have used two decimal fractions which have given a smooth plotting.

Methodology adopted to trace fold profile from photographs has been discussed in Applications part "Procedure" below which two examples of real folds photographs has been brought directly to a spreadsheet and best fit curved has been drawn over them.

Work examples

The shape of the curve can be changed in several ways. As mentioned above, cubic Bézier curve has *four* controlling points, which in Cartesian Coordinates produces eight variables (x_i, y_i): (x_0, y_0) for P_0, (x_1, y_1) for P_1, (x_2, y_2) for $P_2$and (x_3, y_3) for P_3. The coordinates (x_i, y_i) of points P_0 and P_3 are starting and ending points of the curve, respectively. Therefore if the line joining P_0 and P_3 is considered as *base-line*, $x_{base} = x_3 - x_0$ (for fixed y_0 and y_3) determines the horizontal separation between the controlling points P_0 and P_3 which affects the inter-limb angle (Fig. 4a, b). Similarly, $x_{top} = x_2 - x_1$ can be considered as the horizontal distance between P_1 and P_2 which affects the curvature of the limbs. Now by changing the value of x_{base} from 60 to 10, a different set of folds has been generated by keeping P_1 and P_2 separated by $x_{top} = 60$ by fixing them at (0, 60) and (60, 60) (Fig. 4a) or by putting P_1 over P_2 by giving them same coordinate (30, 60) at the middle of the curve (Fig. 4b). In both cases, base-line has been shortened gradually but with different curvature of the limbs. The folds shown in Fig. 4a are semi-circular arcs (when x_{base} is equal to x_{top}) to fan fold (when x_{base} is close to 10) which are ovoid in shape. When x_{base} approaches 0, the shape curves more and resembles to the pendant or Cassini Ovals with $a/b = 1$ (Karataş 2013). On other hand, when $x_{top} = 0$ by putting the same coordinate for P_1 and P_2 at top of center of x_{base}, it is observed that shape of folds transforms from closed parabolic to tight and isoclinal. When x_{base} approaches 0, the fold becomes ptygmatic (Fig. 4b).

On other hand, amplitude (A) of a symmetric fold (as shown in Fig. 4a) depends on coordinates of y_1 and y_2 i.e., $A = y_1 = y_3$. Two tests have been conducted in the *curve*#1 of Fig. 4a and b in which A (by putting $y_0 = y_3$) value is reduced gradually from 60 to 10. As shown in Fig. 4c, the original *curve*#1 of Fig. 4a, in which $x_{top} = 60$ and by decreasing the value of A from 60 to 10, fold shapes varied from semi-circular arc to open and gentle fold. Similarly, from the *curve*#1 of Fig. 4b with $x_{top} = 0$ produced closed parabolic to hyperbolic gentle folds (Fig. 4d).

In Fig. 5, asymmetrical curves has been shown in which P_0, and P_3 are fixed at (0, 0) and (60, 0), respectively, along with fixed $x_1 = 0$ and $x_2 = 60$. In Fig. 5a, only the

Column No. → / Rows No. ↓	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
	-	=(1-A2)^3	=3*(A2)*((1-A2)^2)	=3*(1-A2)*(A2^2)	=A2^3									=(B2*F2)+(C2*G2)+(D2*H2)+(E2*I2)	=(B2*J2)+(C2*K2)+(D2*L2)+(E2*M2)
1	t	B_0	B_1	B_2	B_3	x_0	x_1	x_2	x_3	y_0	y_1	y_1	y_1	$X(t)$	$Y(t)$
2	0.00	1	0	0	0	0	0	10	10	0	10	10	0	0	0
3	0.01	0.970299	0.029403	0.000297	0.000001	0	0	10	10	0	10	10	0	0.00298	0.297
~~															
102	1.00	0	0	0	1	0	0	10	10	0	10	10	0	10	0

fx used

attribute names

In case of a Cubic Bezier Curve, as per eqn (4, 5 and 6) ,
$B_0=(1-t)^3$; Therefore, B_0=> (1-A2)^3
$B_1=3(1-t)^2(t)$; Therefore, B_1=> 3*(A2)*((1-A2)^2)
$B_2=3(1-t)*(t)^3$; Therefore, B_2=> 3*(A2)*(A2^2)
$B_3=(t)^3$; Therefore, B_3=> A2^2
and; where A2 is the first value of column containing t values.
$X(t)= B_0x_0+B_1x_1+B_2x_2+B_3x_3$ and $Y(t)= B_0y_0+B_1y_1+B_2y_2+B_3y_3$

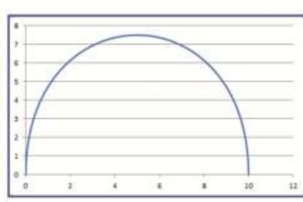

Fig. 3 Layout of the Microsoft Excel spreadsheet showing the arrangement of variables and functions used for plotting of curves on the basis of Eqs. (5) and (6). Columns **A** for $t \in [0,1]$, **B**–**E** are for $B_{i,n}(t)$, **F**–**M** for coordinates of control points P_0, P_1, $P_2$and P_3 and **N**, **o** for $X(t)$ and $Y(t)$. Final plot is done from columns **N** and **O** which is X–Y scattered smooth line graph

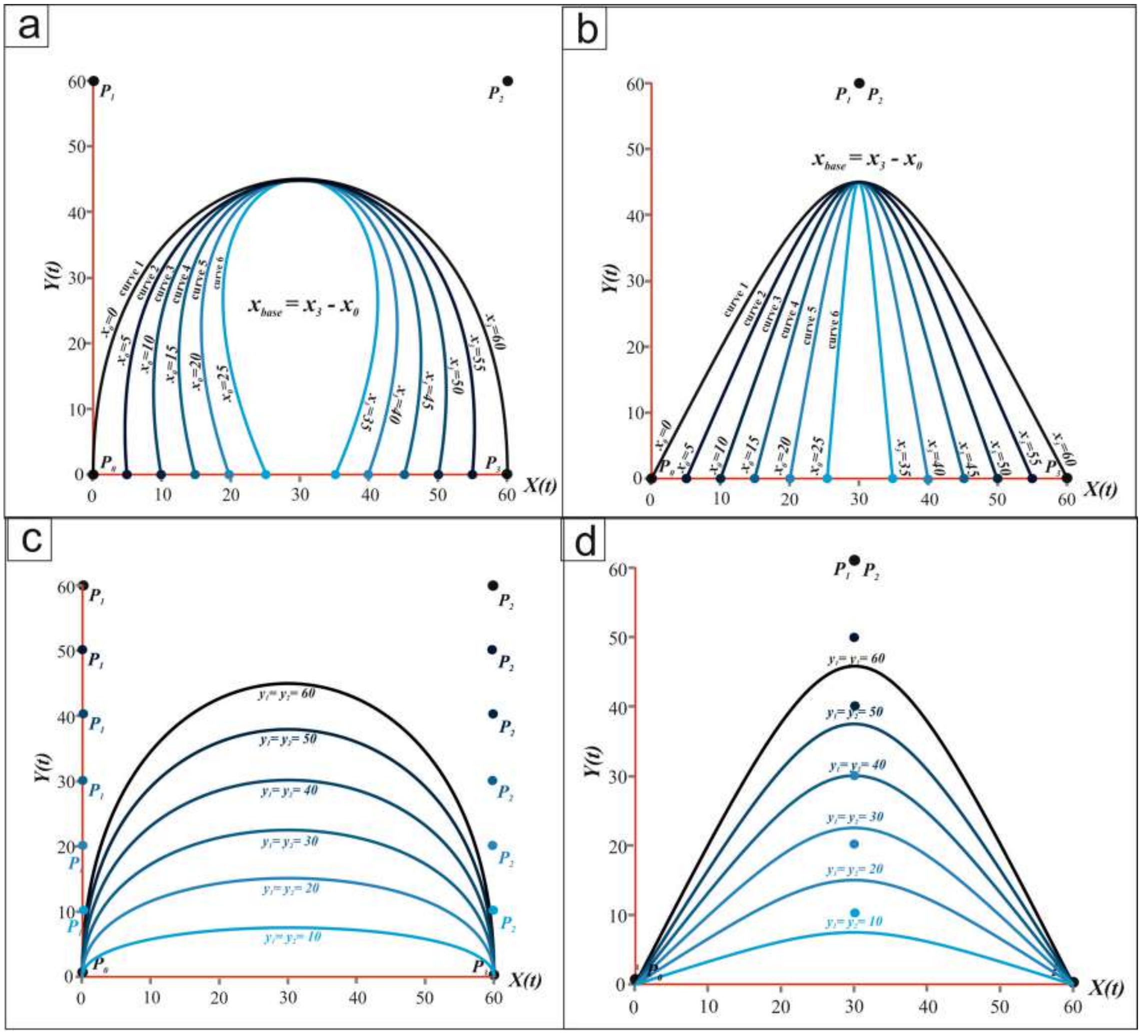

Fig. 4 Cartesian plot showing different cubic Bézier curves developed in the Excel spreadsheet: in (**a**, **b**) control points P_0 and P_3 has been shifted towards middle point of the curve i.e. x_{base} has been changed by shortened from 60 to 10. But in (**a**) control points P_1 and P_2 has been kept apart by keeping x_{top} fixed at 60 and in figure (**b**) both P_1 and P_2 has been kept same at the middle of the curve with x_{top} fixed at 0. In figures (**c**, **d**) the amplitude A has been changed by lowering y_1 and y_2 simultaneously from 60 to 10, but in one case (**c**) x_{top} and x_{base} were kept fixed at 60, and (**d**) P_1 and P_2 has been kept same at the middle of the curve i.e. $x_{top} = 0$

value of y_2 has been changed from 60 to 10 and as shown in Fig. 5b, value of y_1 are altered from 60 to 10. For a fixed horizontal *base-line*, by dropping P_1 and P_2 vertically, left slanted and right slanted curves could be generated respectively. This test suggests that asymmetry of these curves could be dilated with only one variable such as y_1 or y_2 keeping other six variables unchanged. However, several other kinds of curves could also be produced by shifting these control points.

Another test has been conducted on an isoclinal curve which inclination of axial trace increased by shifting the P_1 and P_2 coordinates simultaneously from (10, 60) to (60, 60) horizontally and then vertically to (60, 10) as shown in Fig. 6. In this test, P_1 is put over P_2 and changed simultaneously by giving similar coordinates. It has been observed that the axial trace follows the coordinate of the P_1 or P_2 (as both were taken at the same place). Let $x_c = (x_3 - x_0)/2$ and $y_c = (y_3 - y_0)/2$ be the coordinates of the middle point of control points P_0 and P_3, which could easily be identified (which is 5, 0 in Fig. 6). Let x_a and y_a be the coordinates of P_1 and P_2, therefore, the slope of axial trace could be established from the equation of the straight line (slope–intercept form): $y - y_c = m(x - x_c)$

$$m = \frac{y_a - y_c}{x_a - x_c}.$$

For instance, for the *curve-6* in Fig. 6:

$$m = (60 - 0)/(60 - 5) = 60/55 = 12/11 = 1.09.$$

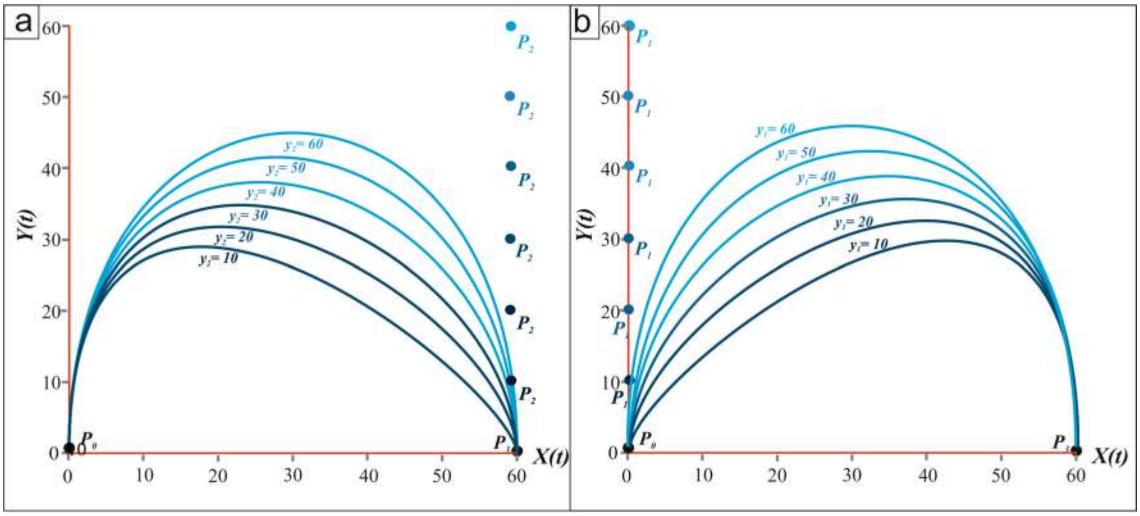

Fig. 5 Sets of slanted curves of varying inclination of axial trance has plotted by changing the values of **a** y_2 from 60 to 10 and **b** y_1 from 60 to 10; keeping rest of the coordinates fixed

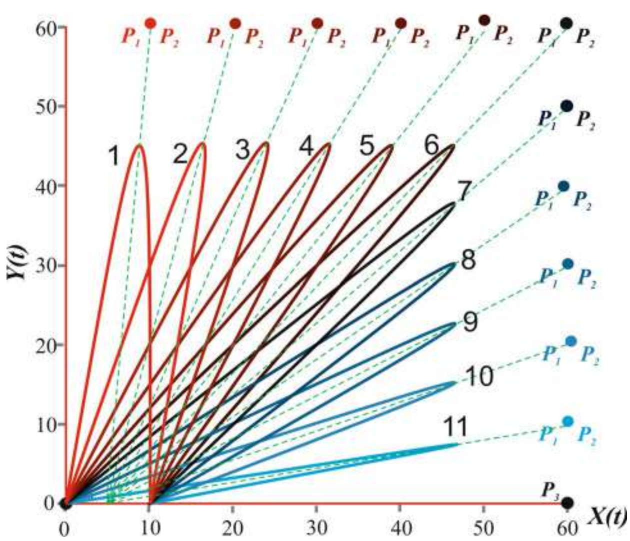

Fig. 6 11 different inclined isoclinal curves has been produced by keeping control points P_1 and P_2 as same point (x_a, y_a) which has been shifted from (10, 60) to (60, 60) and then from (60, 60) to (60, 10). It has been found that axial trace of each curve follows (x_a, y_a). P_0 and P_3 has been kept fixed at (0, 0) and (10, 0), respectively, for the test

Now, let θ be the angle of inclination of axial trace, i.e. $\theta = \tan^{-1}m$.

Therefore, $\theta = \tan^{-1}(1.09) = 47.5°$. This is the angle of inclination of the axial trace of *curve-6*.

Likewise, θ for other curves could also be determined. However, to determine the gradient of an axial trace of the folds, P_1 and P_2 should coincide, as in that case the axial trace will intersect through the point of coincidence and, therefore, we will get two points with known coordinates to determine m and θ or the equation.

When P_0 and P_3 are not in a horizontal line i.e. if $y_0 \neq y_3$, then, $x_{\text{base}} = x_3 - x_0$, may not be the separation distance between P_0 and P_3. In such cases, a distance of separation (D) between P_0 and P_3, following standard equation of line segment:

$$D = \sqrt{(x_3 - x_0)^2 + (y_3 - y_0)^2},$$

e.g. For a curve with P_0 and P_3 located at (0,5) and (15,10), the $D = \sqrt{(15)^2 + (5)^2} = 15.81$.

Procedure

An application of this spreadsheet technique for tracing and re-synthesis of fold has been displayed in Figs. 7 and 8 involving the following steps:

Step 1: Click on the Graph in Excel > Go to Chart Tool > Click on Format Tab > Go to Size Section and note the size of the Graph.

Step 2: Take a photograph of a natural fold in digital format (in JPEG or any supportable format) and resize it inside a frame, which is exactly same as that of the size of the Graph noted in step 1. This will help to trace the fold without altering the aspect ratio of the photograph. We choose Fig. 1.63 of Mukherjee (2015) as an example to demonstrate the process of tracing or re-synthesis.

Step 3: Click on the Graph, go to Chart Tool > Format Tab > Shape Styles Section > Shape Fill > Pic-

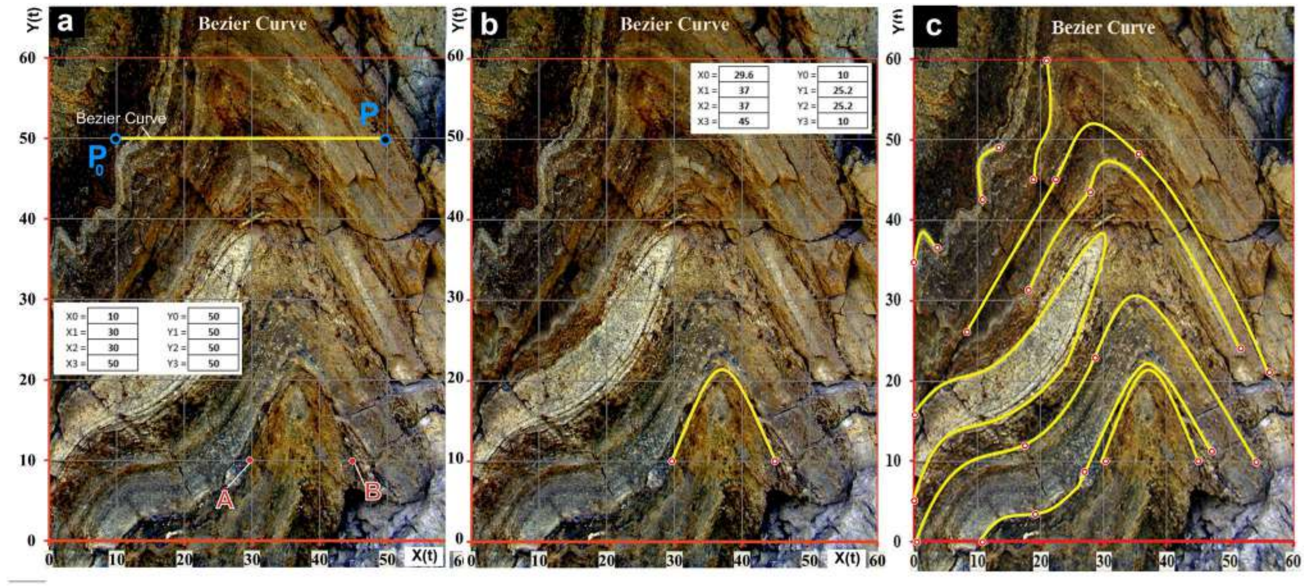

Fig. 7 Steps involved in tracing fold profile in Excel spreadsheet Graph Plot Area from photographs of real example: **a** A photo (Fig. 1.63 of Mukherjee 2015) of real folds has been imported directly as background of Graph Plot Area as given in steps 1 and 2 of Application section. Points A–B of the highlighted (in green dash-line) fold profile has been selected to which the control points P_0 and P_3 of the curve will be brought. **b** By changing the coordinates as shown in table inset, points P_0 and P_3 has been transported to point A and B. Now with little adjustments in coordinates of points P_1 and P_2 the curve has fitted over the fold profile. **c** Some of these layers have been traced in similar procedure with the help of multiple curves. Red dot indicates locations of P_0 and P_3 of these curves

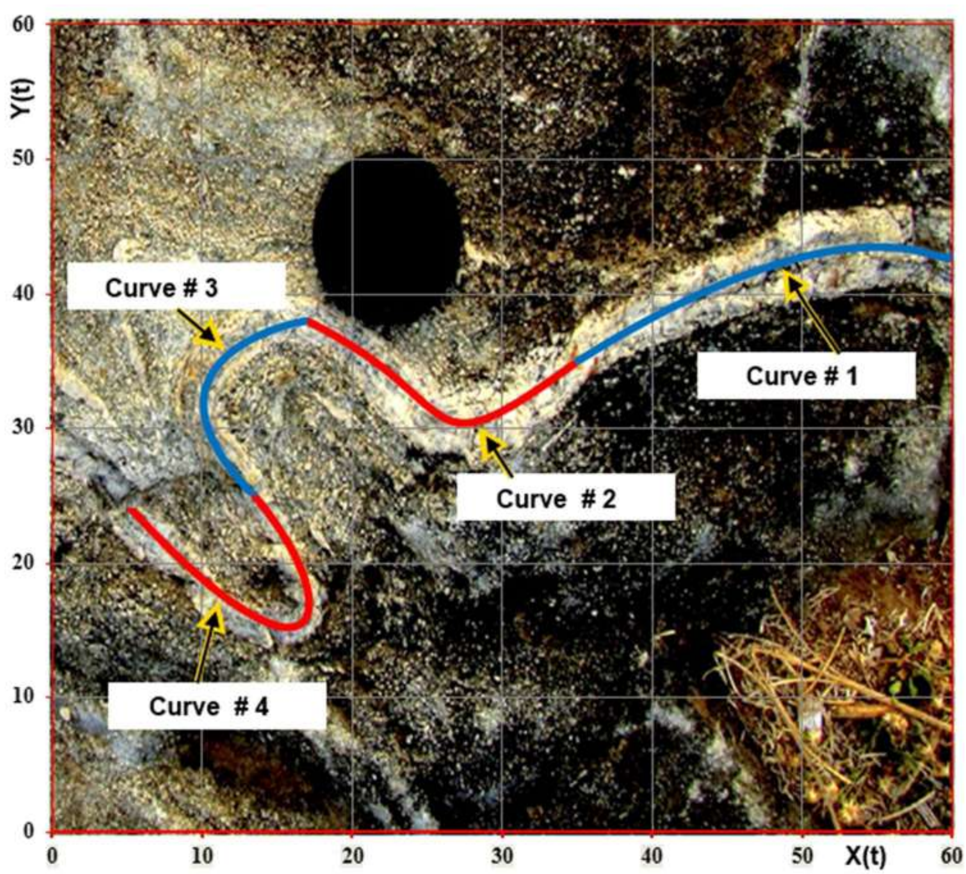

Fig. 8 Another example of tracing by using this technique where the entire folded vein has been traced out in four segments (coordinates given in Table 1). Source of the background photo is Fig. 1.71 of Mukherjee (2015)

ture > import the fold picture. Remove the background colour of Plot Area by clicking on Shape Fill and then No Fill. Adjust the Axes with the edges of the Picture.
Step 4: Activate the Gridlines by clicking on Plot Area > Chart Tool > Layout Tab > Axes Section > Gridlines and chose the suitable gridlines (major or minor).
Step 5: As shown in Fig. 7a, select a fold profile and imagine two points A, B in the fold profile (close to the inflection points) which will be traced. As the grid lines are already activated, now bring up the controlling points P_0, and P_3 near to these points by putting the values of x_0, y_0 and x_3, y_3(Fig. 7b). To match the curve with the fold profile, we will be required to change the values of x_1, y_1 and x_2, y_2 and bring them up just above the crest of the fold. With some adjustments in the values curves will fit above the fold. In Fig. 7c some such curves has been shown that have been traced over these rock layers. Red dot suggests the breaks in curves as some layers have developed multiple folds of different hinges and a single curve cannot cover the entire profile. Therefore, several curves have been used to trace these layers.

Tabulation of data for re-synthesis: In some cases, coordinates of the controlling points are helpful to determine the inclination of the axial trace as mentioned in later part of Results section "Work examples" above or for re-synthesis of the folds. Tabulation of coordinates of a traced curve could be useful to meet such aspects. An example of such tabulation is shown in Table 1, which is helpful to re-plot a curve on a spreadsheet as shown in Fig. 3.

In Fig. 8 tracing of another natural fold taken from Fig. 1.71 of Mukherjee (2015) in four segments (1,2, 3 and 4) has been given. There are four sets of coordinates as listed in Table 1 of the control points, which will give these four-fold segments. It is not possible to trace a non-periodic fold with a single cubic Bézier curve. Therefore, it has been considered in four parts.

Discussions

Advantages of the proposed method

It is not sufficient to describe folds profile by using expressions of sinusoidal waves (sine or cosine), parabolic or hyperbolic equations etc. On the other hand, even though there are certain limitations in working with the Bézier curves, they fit smoothly in most of the fold profiles because of the flexible operation. The method mentioned here is based on the fundamental equation of Bézier curves, the cubic Bézier curve to be specific. The method of synthetic folds generated in this way is simple and can be replicated on any spreadsheet programmes- both commercial and open-source. Note that research works dealing with the simplification of mathematical analyses do exist in other issues of structural geology, such as strain analyses (Chew 2003). The present work uses one of the most commonly used software Microsoft Excel by determining the coordinates of the controlling points (here P_0, P_1, P_2 and P_3) and plotting the values of $X(t)$ and $Y(t)$ putting the values of controlling points in Eqs. (5) and (6) in the Cartesian coordinate system. By importing a digital raster image (*.JPEG, *.GIF etc.) into the Graph Plot Area of the spreadsheet, a segment of folded layer could be traced as discussed in the Application section, and further be re-synthesized by simply putting the coordinates of the earlier fold traced in the same spreadsheet.

The specific advantages are (1) easy re-plotting or re-synthesized of folds either manually or with computer aid, and (2) plotting requires minimum data: only coordinates of the controlling points—x_0, y_0, x_1, y_1, x_2, y_2, x_3 and y_3 to re-synthesize the symmetric and asymmetric folds. Moreover, higher-order Bézier curves could be prepared by expanding the basic equation and can be plotted with more adjustments in the Microsoft Excel sheet. It will help to fit the curves more effectively over fold trains and segmented tracing (as shown in Fig. 8) will not be required.

Table 1 Cartesian coordinates of controlling points P_0, P_1, P_2 and P_3 of four fold segments plotted in the Fig. 8

Sl. no	Photo source	Curve no	Coordinates of controlling points								Remark
			P_0 (starting)		P_1 (intermediate)		P_2 (intermediate)		P_3 (ending)		
			x_0	y_0	x_1	y_1	x_2	y_2	x_3	y_3	
1	Figure 1.71 of Mukherjee (2015)	#1	0	31	17	7	21	15	13.5	25	Total 4 segments, Fig. 8
2		#2	13.5	25	4.5	35	16	38	17	38	
3		#3	17	38	28	32	24	26	35	35	
4		#4	35	35	55	49	60	41	60	43	

Limitations of the proposed method

Since circular arcs cannot be represented by cubic Bézier curves perfectly (Riskus 2006), the present approach cannot give a 100% fit for circular fold profiles. Folds in meso- and micro-scales can have geometries with extrados not matching with its intrados, which are commonly found from deformed migmatites (Mukherjee and Koyi2010). In such a case, none of the existing methods, Bézier and NURB, can uniquely represent the fold profile. Super- and sub-ellipses are to be investigated as the next step to simulate such fold profiles. Folds with straight limbs and sharp hinges (kink folds) are not attempted in this study. Shao and Zhou (1996) attempted a piecewise Bézier curve fitting where a complex algorithm was used to fit a continuous cubic Bézier Curve with irregular curves.

It is also difficult to trace superposed folds or folds with multiple hinges from a single curve. Instead of relying on a single curve, a multi-hinged curve could be manipulated in segments and later joined them digitally. Two or more Bezier curved joined in a series constitutes composite Bézier curves, As in Fig. 8, the curve is actually four curvilinear segments, each being a cubic Bézier curve. They replicate a train of folds. This has already been successfully attempted in the second example in Fig. 8.

Conclusions

This work presents a method of fitting curved to natural folds in 2D using cubic Bézier curves with four controlling points and by utilizing a spreadsheet programme viz., Microsoft Excel. The process is worked out on two field-snaps of folds. Handling a spreadsheet will prove easier for the general geoscientists over the currently practiced computer programs.

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