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Fuzzy set concept in structural geology: Example of ductile simple shear

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Much of the time, geological data are not accurately known. Instead, geoscientists work with a range of possible data. These data can have a range since they can vary spatially and temporally. In this context, we demonstrate how a well-established structural geological model of velocity profile of ductile simple shear, expressed as an equation, can be furthered by fuzzy set concept. The implementation of fuzzy set concepts in terms of fuzzification in (geo)scientific models takes into consideration the uncertainties of magnitudes of the parameters that define the model. The present study shows that the fuzzified velocity profile of a ductile simple shear zone with parallel boundaries is effectively independent of the dip of the shear zone. This article demonstrates how models of (structural) geological processes can be modelled by fuzzy set approach capable of incorporating a range of magnitudes of parameters.

Keywords. Deformation; tectonics; uncertainty.

1. Introduction

Geoscience explains natural phenomena commonly in the abiotic realm (sometimes with biotic inputs) on the Earth and other planets and satellites. For modelling purpose, natural processes such as rock deformation are represented simplistically in terms of equations (e.g., Mukherjee 2019). However, the accurate values of parameters in the model are rarely known to a geoscientist. Rather the known information is a range of magnitudes. One example of such geological data is the age of rock, which is expressed as numbers $x \pm y$. Working with a model equation and datasets will require the concept of fuzzy set theory (e.g., Kahraman *et al.* 2016). In particular, fuzzy set approach has been used increasingly in earthquake hazard studies (e.g., Deyi and Ichikawa 1989), fault slip analysis (e.g., Shan *et al.* 2004), landslide studies (Pradhan 2011), high-resolution mapping (e.g., Bemis *et al.* 2014), reservoir characterization (e.g., Guo *et al.* 2014), ground subsidence (e.g., Park *et al.* 2014), structural complexities at sub-surface (e.g., Justman *et al.* 2020), etc. This article demonstrates how fuzzy set concept can be introduced on an existing structural geological model. Following the same approach, a range of other deformation models can also be fuzzified. The article demonstrates the use of fuzzy set concept in structural geology. The only other interdisciplinary paper in this line is by Justman *et al.* (2020), but that lacks fuzzification of an actual structural geological equation.

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2. Simple shear model

A 'velocity profile' across a shear zone represents the velocity of its particles (e.g., Schlichting and Gersten 1999 in engineering, Mukherjee and Koyi 2010 in structural geology). Mukherjee (2012) presented a general equation (1) of velocity profile (figure 1) of simple shear of an inclined shear zone with very long rigid parallel boundaries full of Newtonian viscous fluid:

$$U_z = 0.5\mu^{-1}[\partial P/\partial z - dg\sin\theta]\left(y^2 - y_0^2\right) + 0.5\{yy_0^{-1}(U_1 + U_2) + (U_1 - U_2)\}. \quad (1)$$

Here U_z is the laminar flow velocity of a Newtonian viscous fluid at (0, y) coordinate (see figure 1 for co-ordinate system), μ is the dynamic viscosity of the fluid, $\partial P/\partial z$ is the pressure gradient that tends to flow the fluid along up-dip direction (positive direction of the z-axis) of the shear zone, d is the density of the fluid, g is the acceleration due to gravity, θ is the dip of the shear zone, U_1 is the shear velocity of the upper boundary along up-dip direction, U_2 is that of the lower boundary acting along the down-dip direction, and $2y_0$ is the orthogonal thickness of the shear zone. Note $y_0 \geq y \geq -y_0$. U_z is defined as per the coordinate system in figure 1. Here the flow is considered to be two-dimensional, i.e., no flow component exists perpendicular to the plane on which the flow is defined.

3. Introduction to fuzzy sets

3.1 *General points*

The uncertain bounds may be modelled using probabilistic methods, interval computations and fuzzy set theory. In probabilistic approach, the uncertain parameters are taken as random variables. On the other hand, in interval and fuzzy computations, the parameters are considered as closed intervals of real line and fuzzy numbers, respectively.

In general, a classical or crisp set S can be defined as a collection of objects or elements of the universal set X. The crisp set assigns a value of either 1 or 0 to each individual object. Whereas, fuzzy set is a class of objects with a grade of membership (characteristic) function, which assigns to each object a grade of membership range between 0 and 1. As such a fuzzy set $\tilde{A}$ may be defined (Zadeh 1965; Zimmerman 2011; Chakraverty *et al.* 2016) as

$$\tilde{A} = \{(x,\ \mu_{\tilde{A}}(x)) :\ x \in X,\ \mu_{\tilde{A}}(x) \in [0, 1]\}, \quad (2)$$

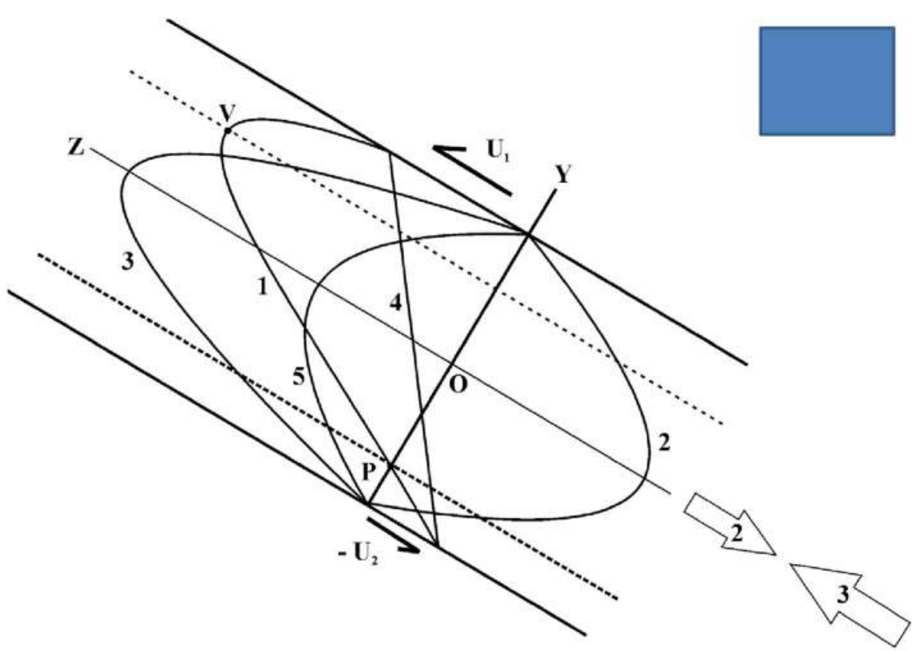

Figure 1. An inclined simple shear zone. Velocity profile 1 is a product of gravity-induced flow (curve 2), simple shear at margins (line 4) and pressure gradient induced flow (curve 3). Curve 5 is a product of resultant flows represented by curves 2 and 3. V: vertex of parabolic profile 1, P: pivot or neutral point of curve 1. Reproduced from figure 1(b) of Mukherjee (2012).

where $\mu_{\tilde{A}}(x)$ is the membership function of the fuzzy set and it is piecewise continuous. A convex and normalized fuzzy set defined on the real line R, whose membership function is piecewise continuous is called a fuzzy number. There are different types of fuzzy numbers namely Triangular Fuzzy Number (TFN), Exponential Fuzzy Number (EFN), Quadratic Fuzzy Number (QFN), and Gaussian Fuzzy Number (GFN) based on the definition of the membership function. The involved uncertainty of the considered problems in the targeted work is considered as fuzzy and the fuzzy numbers are considered as TFN.

3.2 *Triangular fuzzy number (TFN)*

A TFN $\tilde{A} = (a_1, a_2, a_3)$ is a convex normalized fuzzy set $\tilde{A}$ of the real line R such that there exist unique $x_0 \in R$ with $\mu_{\tilde{A}}(x_0) = 1$ (x_0 is called the mean value of $\tilde{A}$) and the membership function of a TFN is defined as (figure 2):

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & x \le a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_2}, & a_2 \le x \le a_3 \\ 0, & x \ge a_3 \end{cases}$$

where $a_2 \ne a_3$ and $a_2 \ne a_1$.

α-cut: TFN $\tilde{A} = (a_1, a_2, a_3)$ may be represented in an interval form by using α-cut:

$$\begin{aligned} \tilde{A} &= \left[\underline{A}(\alpha),\ \bar{A}(\alpha)\right] \\ &= [a_1 + (a_2 - a_1)\alpha,\ a_3 - (a_3 - a_2)\alpha], \alpha \in [0, 1]. \end{aligned} \tag{3}$$

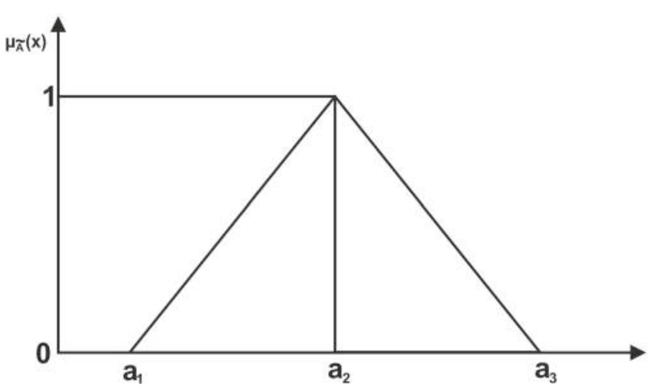

Figure 2. Membership function of a TFN.

3.3 *Fuzzy arithmetic*

For two arbitrary fuzzy numbers $\tilde{A}_1 = \left[\underline{A}_1(\alpha),\ \bar{A}_1(\alpha)\right]$ and $\tilde{A}_2 = \left[\underline{A}_2(\alpha),\ \bar{A}_2(\alpha)\right]$, equality means that $\underline{A}_1(\alpha) = \underline{A}_2(\alpha)$ and $\bar{A}_1(\alpha) = \bar{A}_2(\alpha)$. The arithmetic operations for addition, subtraction, multiplication, and division of fuzzy numbers are defined below:

i) $$\tilde{A}_1 + \tilde{A}_2 = \left[\underline{A}_1(\alpha) + \underline{A}_2(\alpha),\ \bar{A}_1(\alpha) + \bar{A}_2(\alpha)\right], \tag{4}$$

ii) $$\tilde{A}_1 - \tilde{A}_2 = \left[\underline{A}_1(\alpha) - \bar{A}_2(\alpha),\ \bar{A}_1(\alpha) - \underline{A}_2(\alpha)\right], \tag{5}$$

iii) $$k\tilde{A} = \begin{cases} \left[k\underline{A}(\alpha),\ k\bar{A}(\alpha)\right], & k \ge 0 \\ \left[k\bar{A}(\alpha),\ k\underline{A}(\alpha)\right], & k < 0 \end{cases} \tag{6}$$

iv) $$\begin{aligned} \frac{\tilde{A}_1}{\tilde{A}_2} = \Bigg[&\min\left(\frac{\underline{A}_1(\alpha)}{\underline{A}_2(\alpha)}, \frac{\underline{A}_1(\alpha)}{\bar{A}_2(\alpha)}, \frac{\bar{A}_1(\alpha)}{\underline{A}_2(\alpha)}, \frac{\bar{A}_1(\alpha)}{\bar{A}_2(\alpha)}\right), \\ &\max\left(\frac{\underline{A}_1(\alpha)}{\underline{A}_2(\alpha)}, \frac{\underline{A}_1(\alpha)}{\bar{A}_2(\alpha)}, \frac{\bar{A}_1(\alpha)}{\underline{A}_2(\alpha)}, \frac{\bar{A}_1(\alpha)}{\bar{A}_2(\alpha)}\right) \Bigg]. \end{aligned} \tag{7}$$

where $\underline{A}_2(\alpha),\ \bar{A}_2(\alpha) \ne 0$.

4. Structural geological context

In geology, exact values of the six parameters μ, $\partial P/\partial z$, d, θ, $U_i (i = 1, 2)$ in equation (1) are not known that prevailed in the past for any particular simple shear zone. Second, these parameters might have varied over the geological time period. For example, exhuming shear zone material can cool down and increase its μ and d. Depending on the tectonic variation, $\partial P/\partial z$ U_i can vary temporally (Ganguly *et al.* 2000). Shear zones can rotate (Kern and Wenk 1983) and therefore θ can vary. The third uncertainty is that these parameters can be depth-dependent. Density variation with depth can be taken care by taking d = 'representative density' (Mukherjee 2018) in equation (1). In few cases, depth-wise variation of parameters cannot be addressed by fuzzifying equation (1). Such a situation can arise if we consider: (i) variation of θ with depth in case of listric shear zone in orogens (e.g., Xypolias and Kokkalas 2006); (ii) depth-wise variation of μ connoting presumably a non-Newtonian behaviour of the fluid; (iii) vertical depth-dependent $\partial P/\partial z$; and (iv) variation of shear velocities U_i along the boundaries of the shear zones (e.g., stretching faults, Means 1989). In these cases, new equation(s) first of all need to be derived.

We consider here a simple case of time and depth-independent parameters (so that equation

(1) holds) but with their magnitudes not known accurately. In such a case, equation (1) requires fuzzification. A fuzzified equation has the capability to incorporate the range of parameters that the crisp-equation (1) cannot. For a general consideration (i.e., not specific to any particular shear zone), we choose geologically realistic ranges of magnitudes for the six parameters (table 1).

The parameters are used as the Triangular Fuzzy Numbers (TFNs; figure 3a–c). The TFNs were plugged into equation (13). To keep the calculation simple, variables k_i ($i = 1$ to 6) are considered. Equation (14) was solved using TFN arithmetic and the values of the variables k_i were put back into equation (24). Note that the final equation (25) is a function of 'y' so the outputs considered $y = 0$ and 2×10^5 (figure 4a, b). A 3-D plot was made using 'y' as a variable (figure 4c).

For the sake of simplicity, we consider

$$k_1 = 0.5\,\mu^{-1} > 0, \quad \text{since}\,\mu > 0. \tag{8}$$

$$k_3 = g \sin\theta > 0, \text{ for the interval}\, \theta \in [0, \pi/2]. \tag{9}$$

Obviously $k_3 = 0$ for $\theta = 0$, i.e., a horizontal shear zone. As per equation (1), the velocity profile is independent to the density of the material.

$$k_4 = \left(y^2 - y_0^2\right) \le 0 \tag{10}$$

$$k_5 = 0.5 < 0 \tag{11}$$

$$k_6 = y y_0^{-1} > 0, \quad \text{for } y > 0$$

and

$$k_6 = y y_0^{-1} \le 0, \quad \text{for } y \le 0. \tag{12}$$

Putting k_i into equation (1):

$$U_z = k_1(k_2 - dk_3)k_4 + k_5\{k_6(U_1 + U_2) + (U_1 - U_2)\}. \tag{13}$$

Substituting the intervals in equation (13),

$$U_z = k_1 k_4([200,\ 30100,\ 60000] - [2.7, 2.8, 2.9]k_3) + k_5\{k_6([3.17\text{E}{-9}, 1.74\text{E}{-8}, 3.17\text{E}{-8}] + [3.17\text{E}{-9}, 1.74\text{E}{-8}, 3.17\text{E}{-8}]) + ([3.17\text{E}{-9}, 1.74\text{E}{-8}, 3.17\text{E}{-8}] - [3.17\text{E}{-9}, 1.74\text{E}{-8}, 3.17\text{E}{-8}])\}. \tag{14}$$

After simplifying,

$$U_z = k_1 k_4([200, 30100, 60000] - [2.7, 2.8, 2.9]k_3) + k_5\{k_6([6.34\text{E}{-9}, 3.48\text{E}{-8}, 6.34\text{E}{-8}]) + ([-2.85\text{E}{-8}, 0, 2.85\text{E}{-8}])\}. \tag{15}$$

For $y \ge 0$ ($k_6 \ge 0$),

$$U_z = k_1 k_4([200,\ 30100,\ 60000] - [2.7, 2.8, 2.9]k_3) + k_5\{[(6.34\text{E}{-9})k_6 - (2.85\text{E}{-8}),\ (3.48\text{E}{-8})k_6,\ (6.34\text{E}{-8})k_6 + (2.85\text{E}{-8})]\}. \quad (16)\}.$$

Simplifying equation (16),

$$U_z = k_1 k_4([200 - 2.9k_3,\ 30100 - 2.8k_3,\ 60000 - 2.7k_3]) + [((6.34\text{E}{-9})k_6 - (2.85\text{E}{-8}))k_5, ((3.48\text{E}{-8})k_6)k_5, ((6.34\text{E}{-8})k_6 + (2.85\text{E}{-8}))k_5]. \tag{17}$$

Note $k_1 k_4 \le 0$, since $k_4 \le 0$. Therefore,

$$U_z = [k_1 k_4(60000 - 2.7k_3), k_1 k_4(30100 - 2.8k_3), k_1 k_4(200 - 2.9k_3)] + [((6.34\text{E}{-9})k_6 - (2.85\text{E}{-8}))k_5, ((3.48\text{E}{-8})k_6)k_5, ((6.34\text{E}{-8})k_6 + (2.85\text{E}{-8}))k_5] \tag{18}$$

$$U_z = [k_1 k_4(60000 - 2.7k_3) + ((6.34\text{E}{-9})k_6 - (2.85\text{E}{-8}))k_5, k_1 k_4(30100 - 2.8k_3) + ((3.48\text{E}{-8})k_6)k_5, k_1 k_4(200 - 2.9k_3) + ((6.34\text{E}{-8})k_6 + (2.85\text{E}{-8}))k_5]. \tag{19}$$

Table 1. *Chosen parameter ranges for fuzzification of equation (1), taken from Himalayan shear zones in Mukherjee and Mulchrone (2012) and Mukherjee (2013).*

Parameters	Units	TFN fig no.
Viscosity (μ)	10^{18} Pa.s. (i.e., 10^{19} Poise)	None
Pressure gradient ($\partial P/\partial z$)	2–6 Kbar km^{-1} (i.e., 200–60000 Ba cm^{-1})	3(c)
Density (d) of shear zone rock	2.7–2.9 g cm^{-3}	3(b)
Dip (θ) of shear zone	0°–90°	None
Velocity of upper boundary of shear zone (U_1)	1–10 mm y^{-1} (3.17×10^{-9} to 3.17×10^{-8} cm s^{-1})	3(a)
Velocity of lower boundary of shear zone (U_2)	1–10 mm y^{-1} (3.17×10^{-9} to 3.17×10^{-8} cm s^{-1})	3(a)

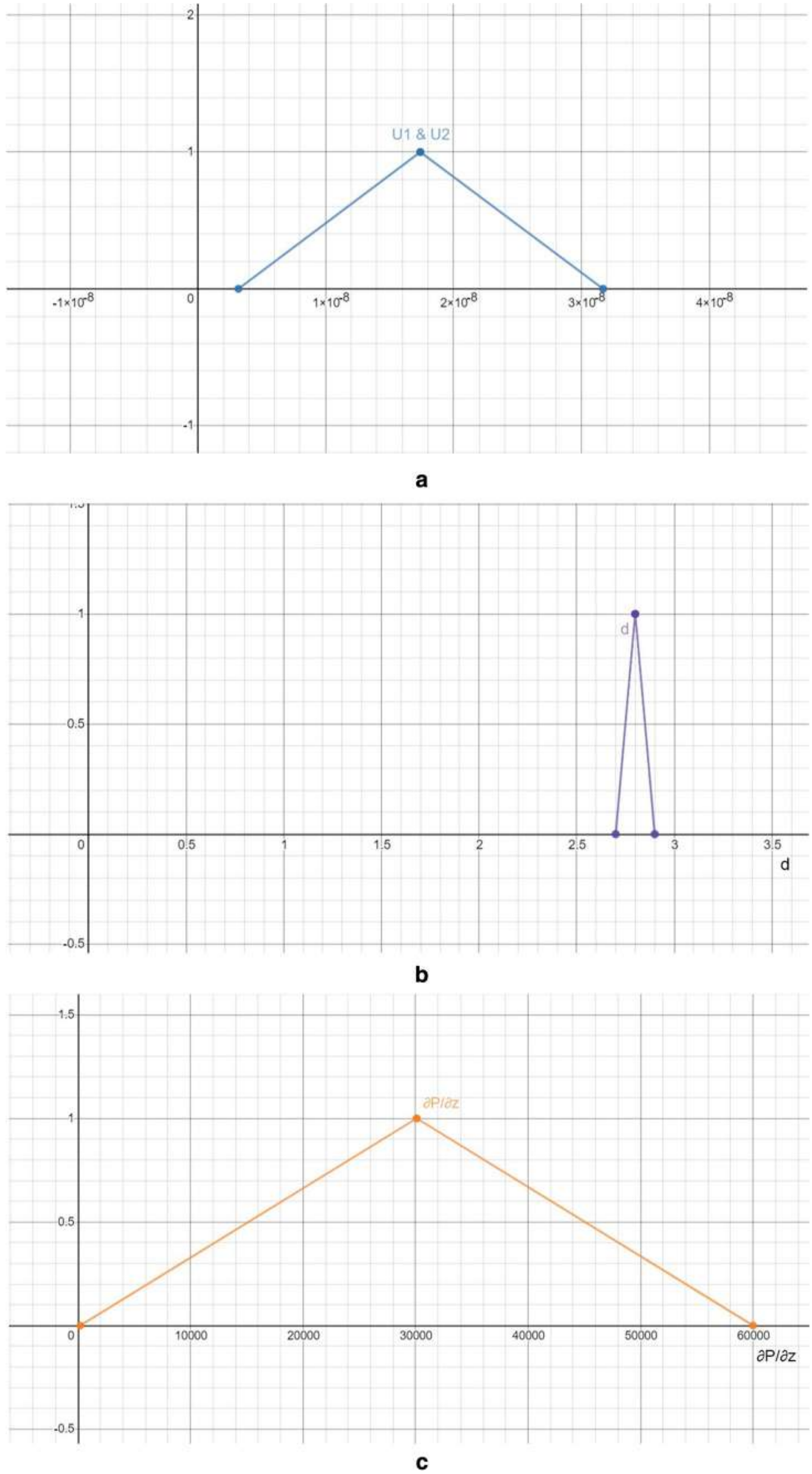

Figure 3. (**a**) TFN: X-axis represents the values of 'U_1' and 'U_2'; Y-axis denotes the membership values. (**b**) TFN: X-axis represents the values of 'd'; Y-axis denotes the membership values. (**c**) TFN: X-axis represents the value of $\partial P / \partial z$; Y-axis denotes the membership values.

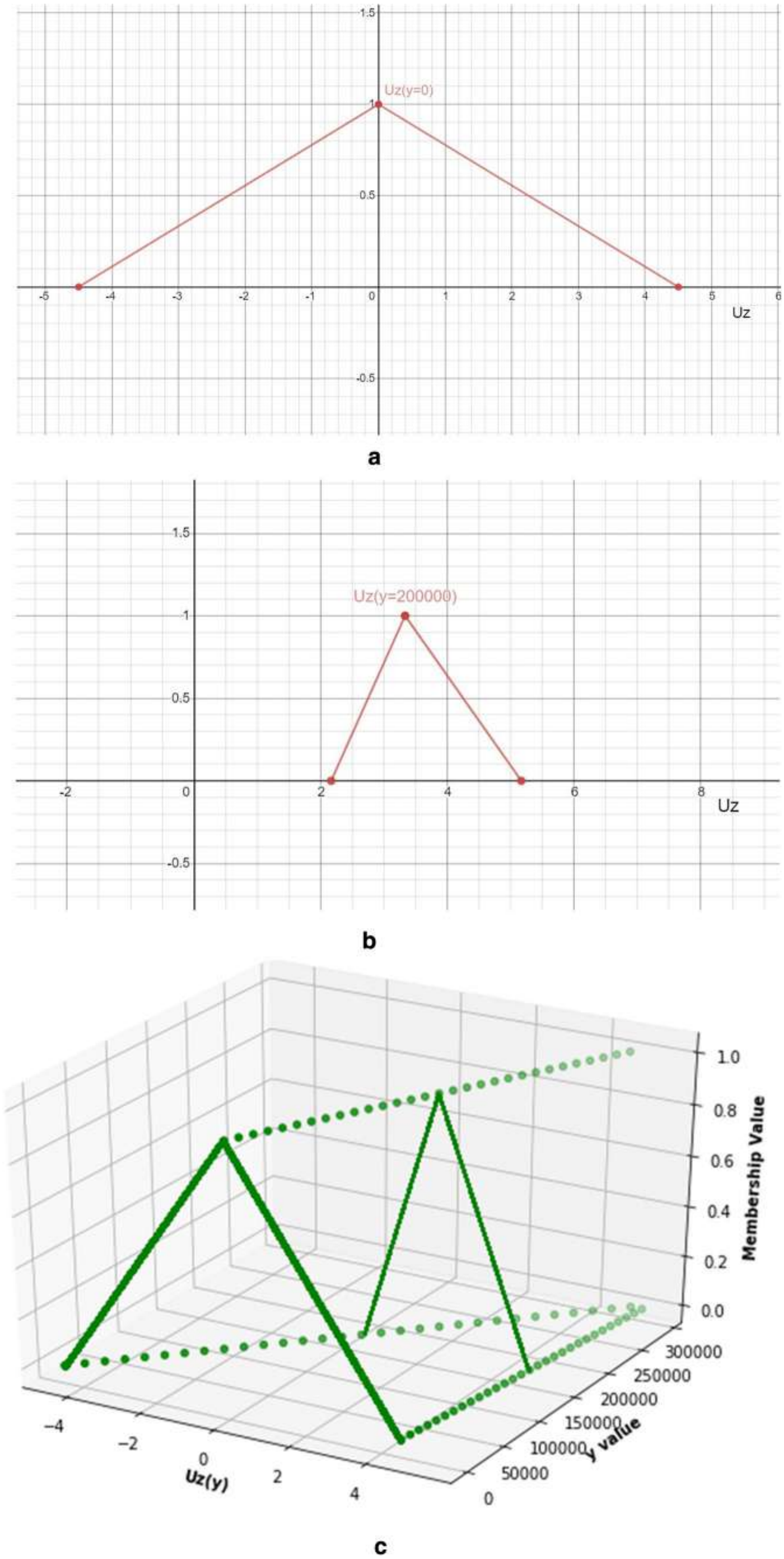

Figure 4. (**a**) TFN: X-axis represents the value of U_z; Y-axis denotes the membership values. (**b**) TFN: For $y = 200000$, X-axis represents U_z; Y-axis denotes the membership values. (**c**) TFN: U_z that has been calculated with the variable y. X-axis represents the value of U_z, Y-axis denotes y (0–300000), Z-axis: membership values.

Inserting the values of the assumed variables in equation (19):

$$U_z = [0.5\mu^{-1}(y^2 - y_0^2)(60000 - 2.7g\sin\theta) + 0.5((6.34\text{E}{-}9)yy_0^{-1} - (2.85\text{E}{-}8)), 0.5\mu^{-1}(y^2 - y_0^2)(30100 - 2.8g\sin\theta) + 0.5((3.48\text{E}{-}8)yy_0^{-1}), 0.5\mu^{-1}(y^2 - y_0^2) \times (200 - 2.9g\sin\theta) + 0.5((6.34\text{E}{-}8)yy_0^{-1} + (2.85\text{E}{-}8))]. \quad (20)$$

Similarly, for $y < 0$ ($k_6 < 0$):

$$U_z = k_1k_4([200,\ 30100,\ 60000] - [2.7,\ 2.8,\ 2.9]k_3) + k_5\{[(6.34\text{E}{-}8)k_6 - (2.85\text{E}{-}8), (3.48\text{E}{-}8)k_6, (6.34\text{E}{-}9)k_6 + (2.85\text{E}{-}8)]\}. \quad (21)$$

$$U_z = k_1k_4([200 - 2.9k_3,\ 30100 - 2.8k_3,\ 60000 - 2.7k_3]) + [((6.34\text{E}{-}8)k_6 - (2.85\text{E}{-}8))k_5, ((3.48\text{E}{-}8)k_6)k_5, ((6.34\text{E}{-}9)k_6 + (2.85\text{E}{-}8))k_5]. \quad (22)$$

We have to quantity $k_1k_4 \le 0$, because $k_4 \le 0$. Therefore,

$$U_z = [k_1k_4(60000 - 2.7k_3), k_1k_4(30100 - 2.8k_3), k_1k_4(200 - 2.9k_3)] + [((6.34\text{E}{-}8)\text{k}_6 - (2.85\text{E}{-}8))k_5, ((3.48\text{E}{-}8)k_6)k_5, ((6.34\text{E}{-}9)k_6 + (2.85\text{E}{-}8))k_5], \quad (23)$$

$$\text{U}_z = [k_1k_4(60000 - 2.7k_3) + ((6.34\text{E}{-}8)k_6 - (2.85\text{E}{-}8))k_5, k_1k_4(30100 - 2.8k_3) + ((3.48\text{E}{-}8)k_6)k_5, k_1k_4(200 - 2.9k_3) + ((6.34\text{E}{-}9)k_6 + (2.85\text{E}{-}8))k_5]. \quad (24)$$

Replacing the values of assumed variables in equation (24),

$$U_z = [0.5\mu^{-1}(y^2 - y_0^2)(60000 - 2.7g\sin\theta) + 0.5((6.34\text{E}{-}8)yy_0^{-1} - (2.85\text{E}{-}8)), 0.5\mu^{-1}(y^2 - y_0^2)(30100 - 2.8g\sin\theta) + 0.5((3.48\text{E}{-}8)yy_0^{-1}), 0.5\mu^{-1}(y^2 - y_0^2)(200 - 2.9g\sin\theta) + 0.5((6.34\text{E}{-}9)yy_0^{-1} + (2.85\text{E}{-}8))]. \quad (25)$$

Figure 4(a) is the TFN plot for equation (25), taking $y = 0$. Note that the output plot for $y = 0$ is symmetric across the Y-axis, so, X = 0 has the maximum membership value. Figure 4(b) presents the TFN plot for equation (25) taking $y = 200{,}000$. We note that the TFN plot shifts to the right side when we increase the value of y and also shrinks in width. Figure 4(c) is the 3D TFN plot for equation (25). For sake of neatness of graph, only a couple of points have been connected using lines. Note that with increasing value of 'y', the plot shrinks and shifts to the right side, i.e., the value of U_z increases. In equation (25), the value of θ does not have a significant effect on the equation, which is counter-intuitive.

5. Discussions and conclusions

The variables and parameters appearing in the geological equations are generally considered as crisp in most formulations. However, the inherent errors in observations and measurement methodologies can bring uncertainty in these parameters, transforming them into fuzzy numbers instead of crisp values. It is a great challenge about how to deal with variables and parameters of uncertain value in these problems. In case of uncertainty, different combinations with interval/fuzzy uncertainty can be considered.

In this work, the aim was to get as close as possible to a specific type of natural ductile simple shear including the range of possible variations of the magnitudes of the parameters. To fuzzify the velocity profile of simple shear of an inclined shear zone, TFNs were used in equation (1). If required, the TFN can later be defuzzified as well. Since the uncertainties were taken into consideration, equation (25) is a better representation of the natural deformation. Through this exercise, it is understood that the natural possible variation of the parameters, viz., density of the shear zone rock, shear velocities at the boundaries and the pressure gradient together can mask the effect of variation of dip of the shear zone in the fuzzified version of the velocity profile.

$$U_z = 0.5\mu^{-1}[\partial P/\partial z - dg\sin\theta](y^2 - y_0^2) + 0.5\ \{yy_0^{-1}(U_1 + U_2) + (U_1 - U_2)\}.$$

In equation (25), variation of dip of the shear zone (θ) from 0° to 90° does not alter significantly the velocity profile. This is because the other shear zone-related parameters, viz., density (d) and viscosity (μ) of rocks, pressure gradient ($\partial P/\partial z$) and the thickness of the shear zone ($2{\cdot}y_0$) have too large magnitudes (table 1) and mask the effect of variation of θ. For example, $[\partial P/\partial z - dg\sin\theta]$

equals $[60000 - 2.7 \times 980 \times \sin\theta]$ in CGS units for $\partial P/\partial z = 6$ Kb km^{-1}, $d = 2.7$ gm cm^{-3} and $g = 980$ cm s^{-2}. For θ within 0° to 90°, the expression varies from 6000 to 3354. Likewise, proceeding as per equation (25) fully, for some specific position in shear zone with respect to the shear zone boundary (or specific y_0 value), one comes across U_z to be practically the same for 0° and 90°. Thus, fuzzification through this work allows us to fully understand how the natural variation of shear zone parameters can affect the velocity profile.

Geological data with uncertainty (Nilsson *et al.* 2006) is quantifiable, and can alternately be tackled by probabilistic models (e.g., Gaines 1978). Probabilistic concept is usually used when a large number of data and associated probability density function, etc., are known for the parameters. But when (i) the parameter values vary in an interval such as the fuzzy numbers, (ii) or interval form with much less number of data, the fuzzy set concept is advantageous to handle uncertainty when the parameter values are known in uncertain but bounded form.

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Author statement

PA: Worked with numericals and in preparation of the draft ms; SC: Supervision, preparation and corrections in the paper; and SM: supervision, writing and finishing the final version of the paper.

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