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Experimental observation of complete and anticipation synchronization of heterogeneous oscillators using a common dynamical environment

Tanu Singla[1,a], E. Chandrasekhar[2], B.P. Singh[1], and P. Parmananda[1]

[1] Department of Physics, Indian Institute of Technology Bombay, Powai, Mumbai-400 076, India
[2] Department of Earth Sciences, Indian Institute of Technology Bombay, Powai, Mumbai-400 076, India

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Abstract. Complete and anticipation synchronization of nonlinear oscillators from different origins is attempted experimentally. This involves coupling these heterogeneous oscillators to a common dynamical environment. Initially, this phenomenon was studied using two parameter mismatched Chua circuits. Subsequently, three different timeseries: a) x variable of the Lorenz oscillator b) the X-component of Earth's magnetic field and c) per-day temperature variation of the Region Santa Cruz in Mumbai, India are environmentally coupled, under the master–slave scenario, with a Chua circuit. Our results indicate that environmental coupling is a potent tool to provoke complete and anticipation synchronization of heterogeneous oscillators from distinct origins.

1 Introduction

Synchronization is a phenomenon in which two interacting systems oscillate with a common rhythm. The first documented experiments on synchronization date back to the ones performed by Christian Huygens wherein he observed that two pendulum clocks, when hung on a common beam, start oscillating in synchrony. The reason for these synchronized oscillations was ascribed to the common mechanical vibrations traveling in the supporting beam. At present, numerous researchers from different backgrounds are engaged in studying synchronization and its distinct manifestations [1–4].

Most of the reported studies on synchronization use an interaction involving a direct communication between the coupled (interacting) oscillators. This interaction can be implemented using instantaneous (identical) variables [5], time delayed (identical) variables [6,7] and time delayed conjugate variables [8]. However, synchronization has also been reported when the uncoupled oscillators are subjected to a common

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[a] e-mail: `tanu@iitb.ac.in`

external forcing [9,10]. Furthermore, results for anticipation synchronization, in the master-slave coupling scenario have been reported employing an interaction term involving a delayed feedback [11].

In the context of systems found in nature, collective dynamics of some oscillators may also occur due to the interaction of these oscillators with their surrounding medium (environment). This implies that, albeit the oscillators do not interact among themselves, they interact with a common surrounding medium. This exchange of information between the oscillators and the common environment may provoke synchronized dynamics in the uncoupled oscillators. For example, in a bacterial population, the bacteria release a signalling molecule (autoinducer) which diffuses into the surrounding medium. The medium senses this molecule and influence the bacteria such that the population exhibits a coordinated activity [12,13]. It has also been reported [14,15], that synchronization of genetic oscillators can be provoked by the diffusion of chemicals between the cells and the extracellular medium. Moreover, it has been documented [16], that collective behavior of the circadian oscillators is invoked by the oscillations in the concentration of neurotransmitters released by these circadian cells. Finally, synchronization of dynamics of the catalysts loaded particles in the Belousov-Zhabotinsky reaction [17] and that of an ensemble of cold atoms in a coherent electromagnetic field [18] occur by virtue of the coupling of these oscillators to their respective common environments. A theoretical framework for synchronization of identical oscillators, by virtue of their coupling with a common dynamical environment, has already been proposed [19,20].

In the present work, we couple heterogeneous oscillators to a common dynamical environment. Results for complete and anticipation synchronization have been observed when the oscillators were coupled, appropriately, to a common environment. The paper is outlined as follows:

In the next section, the coupling mechanism used to couple heterogeneous oscillators to a common environment has been described. This mechanism is similar to the one proposed by Resmi *et al.* [20]. Subsequently, in the same section, we show the experimental Chua circuit designed to implement environmental coupling. In Sect. 3, we show the experimental results for complete and anticipation synchronization. Finally, we summarize the salient observations of the present work in the Conclusions section.

2 Environmental coupling and its experimental implementation

Figure 1 shows two different schemes to couple a pair of oscillators with a common environment. Upper panel of Fig. 1 shows the situation where both the oscillators get influenced by the common environment. The interaction between the micro-organisms in a growing culture medium would fall within the realm of this category. This type of coupling where both the oscillators interact with a common environment is defined as the Bidirectional Environmental Coupling (BEC). In contrast, the lower panel of Fig. 1 shows Unidirectional Environmental Coupling (UEC) scheme. In this scenario, only one of the oscillators (slave) gets influenced by the environment. The other oscillator (master) simply alters the dynamics of the environment without, itself, being affected by it. UEC could be of relevance to efforts involving cloning of the experimental timeseries from different origins.

2.1 Experimental setup

To reiterate, following the mechanism proposed by Resmi *et al.* [20], the coupling of the oscillators, with the common environment involves an exchange of information

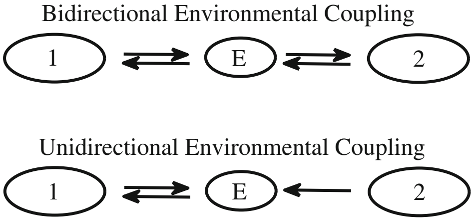

Fig. 1. Schematic diagrams for the two possible scenarios of coupling to a common environment. 1, 2 and E represent the two oscillators and the environment respectively. The upper panel shows the oscillators interacting with the common environment under the Bidirectional Environmental Coupling (BEC) configuration. The lower panel corresponds to the situation where the interaction is of the Unidirectional Environmental Coupling (UEC) type.

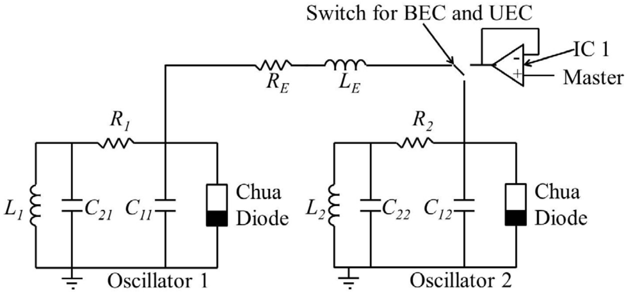

Fig. 2. Experimental setup used to realize environmental coupling. Oscillator 1 and 2 represent the two Chua circuits. Both BEC and UEC can be implemented by changing the position of the switch shown in the figure. In the upper position of the switch the circuit exhibits UEC. On the contrary, in the lower position the circuit exhibits BEC. IC (Integrated Circuit) No. 1 is an Operational amplifier used, in the UEC configuration, as a buffer. Terminal "Master" receives the timeseries from the master oscillator. Output of this IC (master oscillator) couples to the common environment according to the UEC configuration presented in Fig. 1. Here: $L_1 = L_2 = 42.3H$, $C_{11} = C_{12} = 23.5\mu F$, $C_{21} = C_{22} = 235\mu F$, $L_E = 42.3H$. Resistances R_1 and R_2 of the circuit are used to set a parameter mismatch between the two Chua oscillators (used in BEC configuration). R_1 and R_2 were fixed at $1.75\,\mathrm{K}\Omega$ and $1.7\,\mathrm{K}\Omega$ respectively to obtain all the experimental results. The parameter values for the Chua circuit has been obtained from the paper by Tôrres *et al.* [21]. All the results have been acquired by adding the effect of the environment in the potential difference across the capacitor C_{11}/C_{12} of the Chua circuits.

between the oscillators and the common environment. The equations proposed by Resmi *et al.* are:

$$\begin{aligned}\dot{p} &= f_1(p) + \epsilon_1 e\\ \dot{q} &= f_2(q) - \epsilon_1 e \qquad (1)\\ \dot{e} &= -\kappa e - \frac{\epsilon_2}{2}(p - q)\end{aligned}$$

p and q in Eq. (1) represents the dynamics of the individual oscillator. Environment, represented by a scalar variable (e), has κ as its damping constant. Strength of the coupling between the oscillators and the common environment is determined by fixing the values of the constants ϵ_1 and ϵ_2.

Figure 2 shows the schematic diagram of the circuit designed to implement the environmental coupling (BEC and UEC). A simple analysis of the circuit, using the

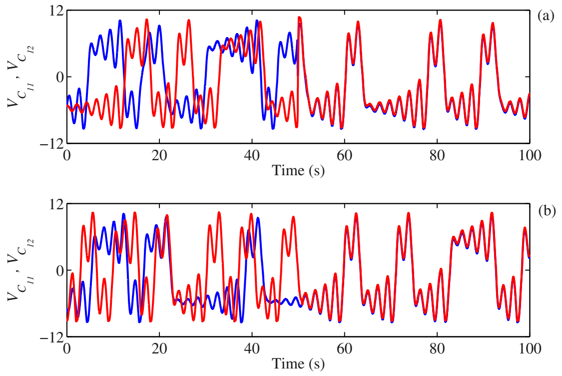

Fig. 3. Experimental results for the synchronization of two Chua oscillators subjected to BEC and UEC. Panel (a) show the timeseries of potential differences across capacitors C_{11} and C_{12} when two Chua circuits were subjected to BEC. Panel (b) show the timeseries of potential differences across capacitors C_{11} and C_{12} when two Chua circuits were subjected to UEC. Blue curve represents the time series of the master oscillator whereas the Red curve represents the time series of the slave oscillator. Coupling was switched on at T = 50 s.

Kirchhoff's voltage law, reveals that the current in the arm containing the resistor (R_E) and the inductor (L_E) is equivalent to the common environment (e in Eq. (1)). Furthermore, it can also be observed that the circuit parameters analogous to the parameters in Eq. (1) are: $\epsilon_1 = 1$, $\epsilon_2 = \frac{1}{L_E}$ and $\kappa = \frac{R_E}{L_E}$. Hence, the coupling between the oscillators and the common environment can be altered by varying the inductor L_E. Since, it is difficult to change the value of the inductor in a continuous fashion, it was fixed at $42.3H$. Therefore, synchronization of the oscillators was achieved by continuously varying the resistor R_E that, in turn, changes the damping constant of the environment. It was observed, that the heterogeneous oscillators synchronized completely for $R_E \approx 600\,\Omega$.

3 Experimental results

3.1 Complete synchronization

In Fig. 3, the synchronization results of two environmentally coupled Chua circuits are presented. Figure 3a corresponds to the superimposed time series of the two nonidentical Chua oscillators coupled in the BEC scenario, whereas, results of Fig. 3b corresponds to the superimposed time series of two nonidentical Chua oscillators coupled in the UEC scenario. Results of Fig. 3 clearly indicate that, it is possible to completely synchronize the dynamics of nonidentical oscillators using environmental coupling.

Figure 4 shows the results when the Chua oscillator (slave) in conjunction with an external timeseries (master) is placed in the UEC configuration. The superimposed timeseries of the Chua oscillator along with the time series of the Lorenz oscillator (upper plot), time series of the Earth's magnetic field (middle plot) and temperature variation signal (lower plot) have been plotted in this figure. It is indeed spectacular to note that, irrespective of the origin and the complexity of the external timeseries, the experimental Chua oscillator is able to mimic the dynamics of the master. For the magnetic field signal employed, a 1-minute averaged time series for the X-component of the Earth's magnetic field was downloaded, and subsequently processed, from the

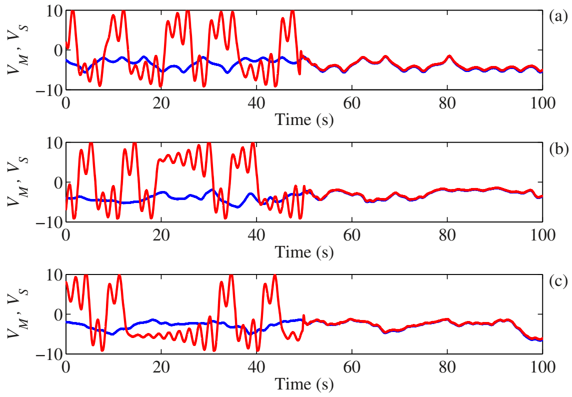

Fig. 4. Experimental results for the synchronization of the Chua circuit with time series of (a) Lorenz oscillator, (b) "X" component of Earth's magnetic field and (c) Temperature variation signal. In all the cases, Blue curves represent the time series of the master oscillators, whereas, the Red curves represent the time series of the slave (Chua circuit) oscillator. Coupling was switched on at T = 50 s.

webpage of ACE (Advanced Composition Explorer) Science Center [22]. The per day temperature variation signal for the Santa Cruz region in Mumbai, India was downloaded from the website of National Climatic Data Center [23]. Please refer to Ref. [24,25] for the relevant information regarding the implementation of UEC for these timeseries.

3.2 Anticipation synchronization

Anticipation synchronization for identical oscillators has already been reported [26,27]. In these published works [26,27], the past dynamics of the slave oscillator were coupled to the present dynamics of the master oscillator to achieve anticipation of the master's dynamics. Furthermore, due to an upper limit for the maximum time by which an oscillator can anticipate the dynamics of the master oscillator, a cascade of slave oscillators coupled in a linear chain had been employed [26,27]. In this chain, each successive oscillator anticipates the dynamics of the previous oscillator resulting in the sequential augmentation of the anticipation time [26,27].

In the present work, the possibility of achieving anticipation synchronization for nonidentical oscillators, under the UEC scenario, is entertained. Subsequently, the cascading strategy [26,27], discussed above, could be implemented to augment this anticipation time.

Figure 5 shows the experimental setup for the single unit of the cascade employed in an effort to anticipate the dynamics of different timeseries using Unidirectional Environmental Coupling (UEC). Similar to the circuit shown in Fig. 2, current in the arm containing resistor R_E and inductor L_E is equivalent to the environment in the coupled system. L_E was fixed at $42.3H$ and R_E was varied to obtain different results. The values of R_E for different results have been given in the respective figures. Delayed dynamics of the slave oscillator (Chua circuit) were generated by using the block "Delay" shown in Fig. 5. Figure 6 gives the detailed description for the delay circuit. The delayed dynamics of the slave oscillator and the master oscillator (Data) are thereafter coupled to the common environment. The effect of the environment is added to the slave oscillator using rest of the circuitry.

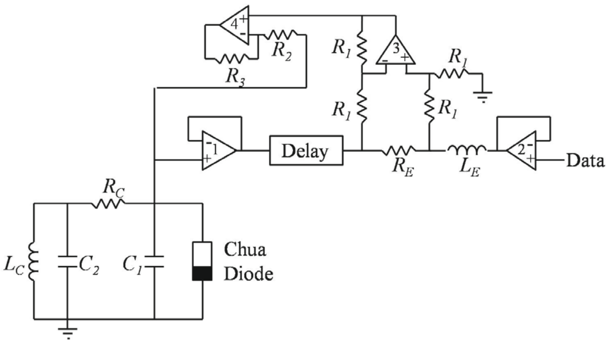

Fig. 5. Experimental setup (single unit) used to anticipate the dynamics of different time-series using the Unidirectional Environmental Coupling (UEC). Chua circuit was used as the slave oscillator and the time series of the master oscillator was connected at the terminal "Data" of the unit. IC No. 1-4 are simple operational amplifiers. The values of the components of the Chua circuit and for the environment were same as given in Fig. 2. IC No. 3 was used as the difference amplifier. It calculates a term directly proportional to the current (environment) through resistor R_E. The environment is then coupled to the slave oscillator using IC No. 4. The values of the remaining components were: $R_1 = 1\,\text{K}\Omega$, $R_2 = 1.2\,\text{K}\Omega$ and $R_3 = 3.3\,\text{K}\Omega$.

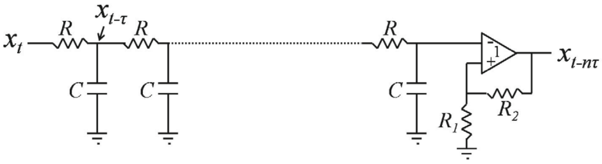

Fig. 6. Experimental setup of the circuit used to generate the delayed dynamics of an oscillator. The circuit consists of a series combination of RC networks. For an input x_t at one end of the RC network, potential difference across the capacitor (C) is delayed by a time period of $\tau = RC$. Connecting n such units in series will generate a delay of $n\tau$ at the output of the Op-amp (IC No. 1). Values of the resistance and capacitance used in the circuit were: $R = 1\,\text{K}\Omega$ and $C = 100\,\mu F$. For our results, three such RC networks were used in one delay circuit. Hence, the maximum delay that the delay circuit can produce was 300 ms.

Figure 7a depicts the anticipation for the time series of a Chua circuit. For these results, a time series from a Chua circuit was generated and processed [24,25] to couple it with the environment. This time series (master oscillator) was connected at the terminal "Data" of the first unit (first slave) of the cascade used for anticipation. As shown in Fig. 7a, Black curve represents the master oscillator and the Blue curve represents the timeseries of the first slave. Since, the anticipation time between the first slave and the master oscillator was not significant, the time series of the first slave was then connected to the terminal "Data" of the second unit (second slave) of the cascade. This process was further repeated for the third slave. Red and Green curves in Fig. 7 represent the time series of the second and the third slave of the cascade respectively. It is observed that, using three slaves to anticipate the dynamics of a Chua oscillator results in an enhancement of the anticipation time.

The anticipation time mentioned in the caption of Fig. 7 was obtained by calculating the similarity function ($S(\tau)$) between the master oscillator and the third slave.

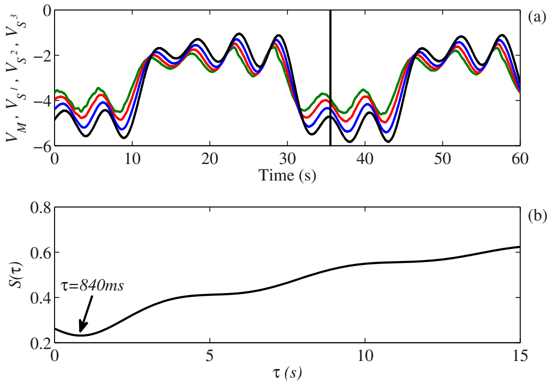

Fig. 7. Results for the anticipation of a timeseries for the Chua circuit ($R_E = 600\,\Omega$). (a) Black curve represents the timeseries of master oscillator. Blue, Red and Green curves respectively represent the time series of first, second and the third slave. (b) Similarity function between the present of the master oscillator (V_M) and the past of the third slave (V_{S_3}). Anticipation time (measured from the similarity function) between the master oscillator and the third slave was measured to be $\tau \approx 840\,\text{ms}$.

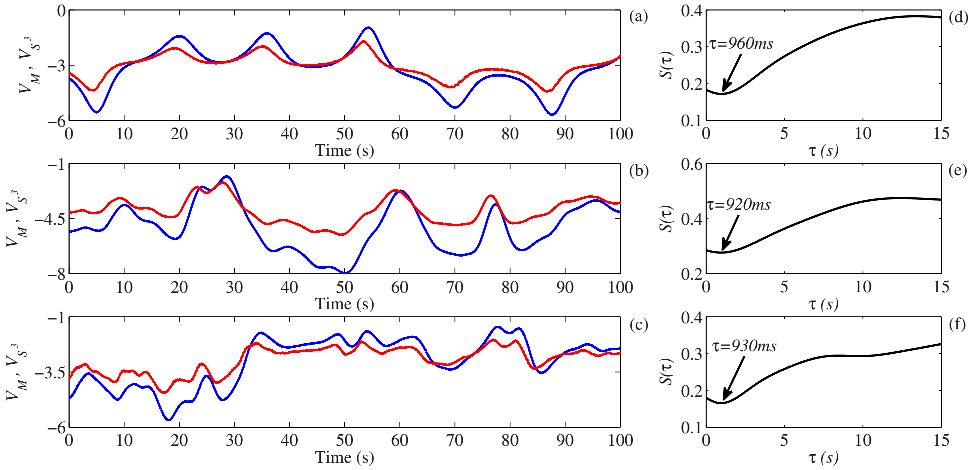

Fig. 8. Results for anticipation of timeseries of (a) Lorenz oscillator ($R_E = 600\,\Omega$), (b) X-component of Earth's magnetic field ($R_E = 600\,\Omega$) and (c) temperature variation signal ($R_E = 800\,\Omega$). Blue curves represent the timeseries of the master oscillators/timeseries and the Red curves represent the time series of the third slave for the different cases. Panels (d–f) show the plots of similarity function for Lorenz oscillator, Earth's magnetic field and the temperature variation signal respectively. For the Lorenz oscillator, anticipation time between master and third slave was $\tau \approx 960\,\text{ms}$ and that for magnetic field and temperature variation was $\tau \approx 920\,\text{ms}$ and $\tau \approx 930\,\text{ms}$ respectively.

Similarity function used to calculate the anticipation time can be defined as:

$$S(\tau) = \left(\frac{\langle [V_M(t) - V_{S_3}(t-\tau)]^2 \rangle}{[\langle V_M^2(t) \rangle \langle V_{S_3}^2(t) \rangle]^{1/2}} \right)^{1/2}$$

Figure 7b shows the plot of the similarity function between the master oscillator and the third slave. The minimum value of S(τ) at $\tau = 840\,\text{ms}$ confirms anticipation synchronization between two Chua circuits coupled with a common environment. $\tau = 840\,\text{ms}$ is the anticipation time between the two Chua circuits.

Furthermore, since it is possible to synchronize Chua circuit with the time series from different origins (Fig. 4), an attempt was made to anticipate these oscillators using the strategy explained in the paragraph above. Results for anticipation of oscillators from different origins have been shown in Fig. 8. Panel (a) shows the time series of the Lorenz oscillator (Blue curve) and the third slave oscillator (Red curve). Similarly, panels (b) and (c) show the results for Earth's magnetic field and temperature variation signal respectively [24,25]. Panels (d–f) show the similarity function corresponding to the time series shown in panels (a–c). The anticipation time for various oscillators is mentioned in the caption of Fig. 8. For magnetic field signal, $\tau = 920\,\text{ms}$ corresponds to an anticipation of 13 mins. However, $\tau = 930\,\text{ms}$ for the temperature variation signal corresponds to an anticipation of 2 days.

4 Conclusions

Our novel experimental results indicate that it is indeed possible to achieve complete synchronization of heterogeneous oscillators using environmental coupling. To reiterate, BEC of nonidentical oscillators could provide an insight into situations where oscillators interact with their common environment and can exhibit coordinated collective dynamics. Furthermore, coupling of the Chua oscillator and the external timeseries with the common environment, under the UEC scenario, resulted in the synchronization of the slave (Chua) oscillator with the external timeseries from different origins. This mechanism could prove useful in efforts involving modelling of real systems. Finally, UEC is also successful, when used appropriately, in anticipating the timeseries of oscillators from different origins. The time of anticipation for the oscillators can be sequentially enhanced by coupling the slave oscillators in a linear chain. Further experiments in different physical, chemical and biological systems need to be carried out to validate the robustness, universality and hence the applicability of our experimental results.

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24. Since discreet data-sets are used in the experimental observations shown in Figs. 4, 6 and 7, their synchronization with the Chua circuit depends on the time interval between two consecutive data points fed to the common environment. For results in Figs. 4 and 7 this time interval was maintained at 200 ms for Lorenz oscillator, 66.6 ms for Earth's magnetic field and 333.3 ms for the temperature variation signal. However, for timeseries of Chua oscillator in Fig. 6, this time interval was maintained at 32 ms
25. It needs to be pointed out, that the timeseries of oscillators used in Figs. 4, 6 and 7 were transformed for the experiments due to technical considerations. As can be seen in these figures, using a linearly transformation, the timeseries were restricted to negative values
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