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Multifractal Analysis of Geomagnetic Storms

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Abstract

Multifractal detrended fluctuation analysis (MFDFA) has been applied to a few geomagnetic storms of solar cycle 23, recorded at Kakioka magnetic observatory to understand their multifractal behaviour. The multifractal singularity spectra of main phases of March 19 and April 21 storms (set-1) and those of March 27 and October 21 (set-2) clearly illustrates that set-2 shows a relatively greater degree of multifractality than set-1. The November 06 event shows the highest degree of multifractality among all storms. This is believed to be due to the prompt penetration of magnetospheric electric fields to low latitudes during this event. The respective recovery phases of all the storms show similar behaviour, albeit with lower degree of multifractality. The multifractal behaviour of storms is determined to be due to the presence of long-range correlations in the data. MFDFA of solar quiet day (April 30, 2001) shows an almost monofractal behaviour.

1 Introduction

The science of geomagnetism that explains the morphological processes responsible for various types of geomagnetic activity, ranging from a few seconds to hundreds of years is fascinating. Among different types of geomagnetic phenomena, the geomagnetic storms in particular, are very widely studied events of extra-terrestrial origin that are still being probed for better understanding of the Earth's external and internal magnetic environment. Although the coronal mass ejections (CME) and subsequent ring current formation are the prime causes for occurrence of geomagnetic storms (Gosling et al., 1990; Daglis et al., 1999), and although the magnetospheric dynamics that occur during the evolution of these magnetic storms is highly stochastic and non-deterministic, it is intriguing to observe that the morphology of all storms is same. However, there appears to be some order in such chaotic phenomena, which can be better visualized by studying their spatio-temporal characteristics and multifractal behaviour. Therefore, a deeper understanding of such well-known chaotic sources is always important for better characterization of the magnetospheric dynamics.

Novel signal analysis techniques such as wavelet analysis, fractal and multifractal studies, etc., aid in effective characterization of the upper atmospheric phenomena. It has long been realized that geomagnetic field of extra-terrestrial origin, particularly, the geomagnetic storms exhibit statistical self-affinity properties (Uritsky et al., 2001; Sitnov et al., 2001; Kovacs et al., 2001; Liu, 2002; Wanliss, 2005; Balasis et al., 2006, to cite a few). Zaourar et al (2013) made an explicit study of wavelet-based multiscale analysis of geomagnetic disturbance and characterized different phases of magnetic storms using the scaling exponent as the diagnostic tool.

While Peng et al (1994) popularized the fractal studies using detrended fluctuation analysis (DFA), which has found its applications in a vast majority of fields, Kantelhardt et al (2002) provided a detailed formalism for multifractal DFA (MFDFA), which is gaining its importance in improved characterization of stochastic signals in a broader perspective. In the present study, we make an attempt to study and compare the multifractal behaviour of geomagnetic data of solar quiet and disturbed days of March, April, October and November of the year 2001, corresponding to solar cycle 23 using MFDFA technique. MFDFA facilitates to determine the Hurst exponents and multifractal singularity spectra to understand the multifractal behaviour of the signals. We also discuss whether the existence of multifractality in the data is either due to the presence of long range correlation or broad probability distribution. In the following two sections, we briefly describe the data and methodology of MFDFA technique. Next, we discuss the results, discussion and conclusions.

2 Data base and processing

For the present study, the geomagnetic horizontal north-south (H-) component data sets of geomagnetic storms and solar quiet (Sq) days sampled at 1-sec interval, recorded at the Kakioka (IAGA code: KAK) magnetic observatory have been procured from world data centre, WDC-C2, Kyoto, Japan. Data corresponding to magnetic storms were identified based on Dst index and those of quiet days were selected corresponding to Ap ≤ 6 (see Chandrasekhar et al., 2003). A few intermittent jumps, missing values, etc., observed in the data were corrected prior to further analysis. The data quality of KAK observatory was very good and therefore, not much data processing was needed to be done prior to further analysis.

3 MFDFA Formulation and Methodology

3.1. Formulation

For estimation of scaling exponents, the signal under investigation must be unbounded, which can be converted to a self-similar process by integration (Goldberger et al., 2000). The thus generated integrated series is divided into short windows (of equal length) and the average fluctuations associated with each window of data are obtained by calculating the trend (a least-squares fit) of the data in each selected window and removing it from each data point of the corresponding window. This is repeated for various window lengths of data. The MFDFA describes a generalized form of DFA (see Peng et al (1994) for full details of DFA process), in which, the different orders of fluctuation functions (also known as moments) are estimated in a modified least-squares sense. In MFDFA (unlike in DFA), in order to account for the left-over points at the end of data series for any chosen window length, the average fluctuations are calculated in both forward and backward directions and averaged. The multifractal Hurst exponents are then determined from the slopes of the linear least-squares regression between the logarithm of overall average fluctuations and the logarithm of lengths of the windows, corresponding to different orders of fluctuation functions. By establishing a relation between the Hurst exponent and Hölder exponent (see details below), the multifractal singularity spectrum is defined. Mathematical description of MFDFA technique is as follows.

First generate an integrated series $y(m)$ of N -point spatial data sequence, say, $x(s)$, by estimating $y(m)=\sum_{i=1}^{m}\left(x(i)-\bar{x}\right) \quad ; \quad m=1,2,3,\ldots\ldots\ldots\ldots N$, where, $\bar{x}$ designates the mean of the N data points. Next, divide the *m*-length integrated series into various m/k non-overlapping windows (N_k) of equal length and calculate the least-squares fit of preferred order to the data points in each window to represent the local trend y_k. Calculate the average fluctuations, $F(k,n)$, of the detrended series in forward and backward directions by $F(k,n)=\sqrt{\frac{1}{k}\sum_{i=s+1}^{nk}\left[y(i)-y_k(i)\right]^2}$ where, $s=(n-1)k$ and n depicts the window number. The generalized form of overall average fluctuations of q^{th} order fluctuation function is expressed as $F_q(k)=\left\{\frac{1}{2N_k}\sum_{n=1}^{2N_k}\left[F^2(k,n)\right]^{q/2}\right\}^{1/q}$ (Kantelhardt et al., 2002), which is iteratively calculated for various q and window lengths, k to provide a power-law relation between $F_q(k)$ and $k^{h(q)}$. The multifractal Hurst exponent $h(q)$ defines the slope of the linear least-squares regression between the logarithm of the overall average fluctuations $F_q(k)$ and the logarithm of the window length k, for corresponding q. Fluctuation function of 0$^{\text{th}}$ order is calculated using the formula, $F_0(k)=\exp\left[\frac{1}{4N_k}\sum_{n=1}^{2N_k}\ln\left\{F^2(k,n)\right\}\right]\approx k^{h(0)}$. The multifractal singularity spectrum, defining the relation between the singularity spectrum, $f(\alpha)$ and strength of the singularity, α (Hölder exponent), is given by $\alpha = h(q) + qh'(q)$ and $f(\alpha) = q[\alpha - h(q)] + 1$ (see Kantelhardt et al (2002) for more details on α and $f(\alpha)$). Singularity spectrum resembles the Gaussian shape. Broad (narrow) singularity spectrum signifies strong (weak) multifractal behaviour of the signal (Bolzan et al., 2013).

3.2 Methodology

Geomagnetic horizontal north-south (H-) component of all the storms, sampled at 1-sec interval have been subjected to MFDFA technique, described above. To determine the multifractal Hurst exponents for different moments q, the minimum window length varying in the range 10 s – 20 s was used. The maximum window length is $N/4$ s. N denotes the total number of data points in each storm, which varies according to the duration of storm activity. The successive window lengths are incremented by a factor of $2^{1/8}$ (Peng et al., 1994). The multifractal Hurst exponents, $h(q)$, were determined by varying q in the range, -10 to 10 for all the storms. Figures 1 and 2 depict the Hurst exponents of main phases (Fig. 1a) and recovery phases (Fig. 2a) of all the storms. The respective singularity spectra are shown in Fig. 1b and Fig. 2b respectively.

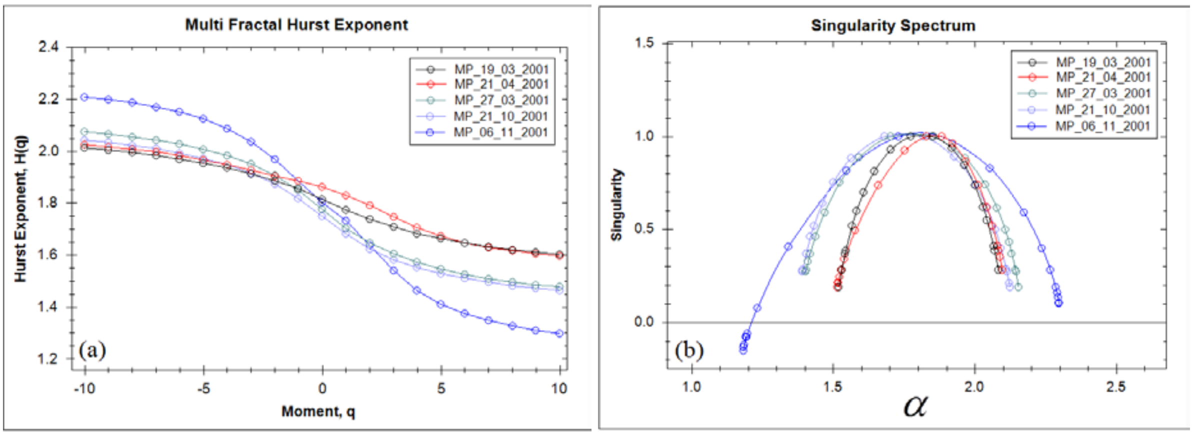

Figure 1: (a) Hurst exponents and (b) Singularity spectra curves of main phase for geomagnetic storms considered in the present study.

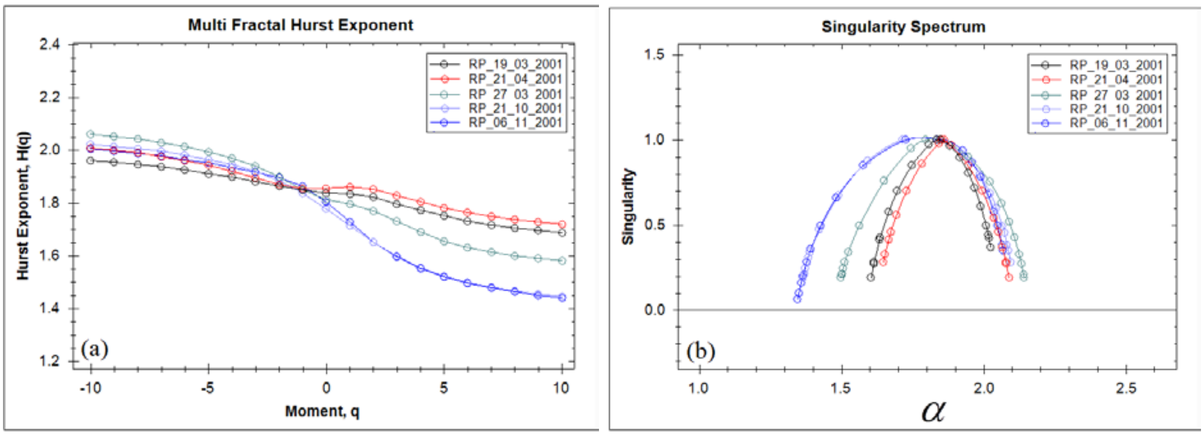

Figure 2: (a) Hurst exponent and (b) Singularity spectra curves of recovery phase for 2001 geomagnetic storms considered in the present study.

To compare the multifractal behaviour of external sources of different origins, the MFDFA of one quiet day (April 30, 2001) has been carried out. In case of quiet days also the minimum and maximum window lengths are chosen similar to those of storms. Figure 3 depicts the Hurst exponent (Fig. 3a) and the multifractal singularity spectra (Fig. 3b) corresponding to the chosen quiet day.

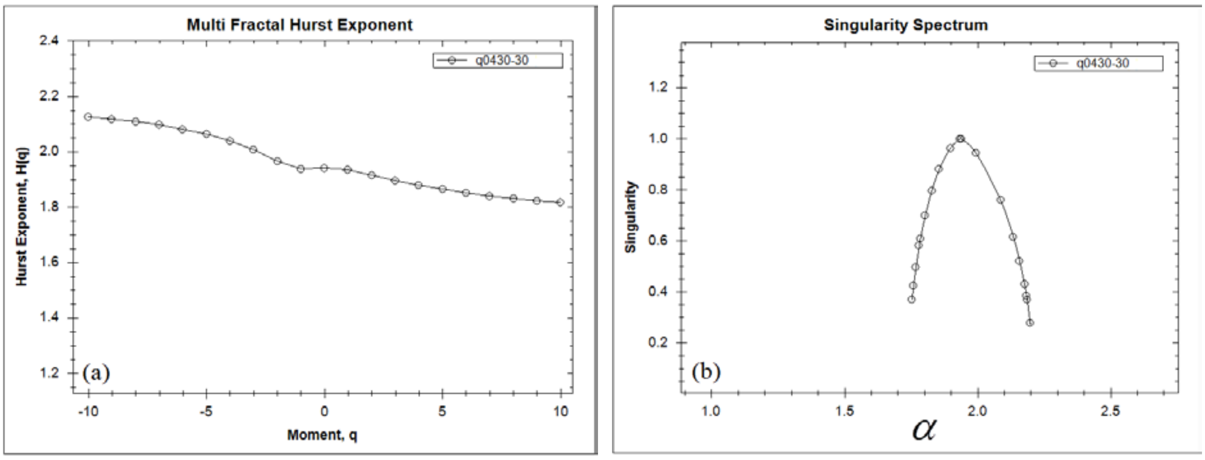

Figure 3: Hurst exponent (a) and Singularity spectrum (b) of quiet day (April 30, 2001). Note the negligible changes in Hurst exponent values as a function of q and a very narrow singularity spectrum as a function of α, suggesting an almost monofractal behaviour of quiet days signifying the probable non-stochastic behaviour of the ionospheric source currents responsible for generation of quiet days.

4 Results and Discussion

The multifractal behaviour of any signal is generally interpreted by considering the Hurst exponents together with the multifractal singularity. The Hurst exponent, $h(q)$ bears a non-linear relation with q, such that the low (high) average fluctuations have high (low) $h(q)$ for negative (positive) q (Kantelhardt, 2002). If $h(q)$ varies (constant) for various q, then the signal exhibits multifractal (monofractal) behaviour. Accordingly, the main and recovery phases of geomagnetic storms exhibit multifractal behaviour with the Hurst exponent values ranging from 1.3-2.2 for main phase (Fig. 1a) and 1.45-2.05 for recovery phase (Fig. 2a). Based on the broadness of the singularity spectra (cf. Bolzan et al., 2013) of main phase (Fig. 1b) and recovery phase (Fig. 2b), it can be easily argued that the former exhibits slightly broader spectra than the latter, suggesting a higher degree of multifractality in the main phase than in the recovery phase. This has been the nature of the multifractal behaviour for all the storms considered in the present study.

A careful observation of Figures 1 and 2 clearly show that while the main phase and recovery phases of storms of March, April and October show almost similar multifractal behaviour, the singularity spectra of main phase and recovery phase of November 06 event (occurred in relatively less intense winter season) are the broadest when compared to respective phases of other storms, indicating a highest degree of multifractal behaviour of the source corresponding to this event. Accordingly, the Hurst exponents of respective phases also show a large variation, when q varies from -10 to 10. It is believed that the prompt penetration of magnetospheric electric fields to low latitudes during the November 06 event (Veenadhari et al., 2010) could be the reason for such high degree of multifractal behaviour of November 06 storm. The almost similar behaviour of the morphology of the source corresponding to March 19 & April 21 pair of storms and March 27 & October 21 pair of storms clearly suggest the presence of some order in their chaotic and stochastic source. Interestingly, these four storm events correspond to equinoctial (E-) season. To have a thorough and comprehensive understanding of the seasonal dependence of the multifractal behaviour of the source morphology of different storms, more storm events should be analyzed.

Interestingly, corresponding to a quiet day (April 30, 2001), the changes in Hurst exponent values as a function of q is rather very small (Fig. 3a), and remains almost constant. The corresponding singlularity spectrum is also very narrow (Fig. 3b), suggesting an almost monofractal behaviour of the ionospheric source current systems during solar quiet days.

Generally, the multifractality in any signal may arise either due to the presence of long-range correlations in the data or due to the broad probability distribution (Kantelhardt et al., 2002). This can be tested by shuffling the original data and comparing the scaling exponents obtained for the original and shuffled data. When the original data is shuffled, it becomes uncorrelated random noise. If the fractal scaling exponent (estimated for $q = 2$) for the shuffled data (i.e. uncorrelated random noise) is equal to 0.5, then the observed correlations are due to long-range correlations present in the original data. Otherwise (i.e. if the scaling exponent is not equal to 0.5), the observed correlations are due to the broad probability distribution in the data (Goldberger et al., 2000). This has been tested for all storms in the present study. Table 1 shows the scaling exponent values corresponding to main phase and recovery phases of all storms. A careful observation of the fractal scaling exponents of shuffled time series of all the storms clearly suggest that the observed multifractal behaviour in all the storms is solely due to the presence of long-range correlations in the data and not due to the broad probability distribution.

Storm date	Phase of the storm	$h(2)$	$h_{\text{shuffled}}(2)$
March 19-21	Main phase	1.74	0.51
	Recovery phase	1.82	0.51
March 27-28	Main phase	1.65	0.51
	Recovery phase	1.77	0.49
April 21-23	Main phase	1.80	0.49
	Recovery phase	1.86	0.50
October 21-23	Main phase	1.62	0.50
	Recovery phase	1.65	0.50
November 06-07	Main phase	1.64	0.48
	Recovery phase	1.66	0.49

Table 1: Fractal scaling exponents, calculated for fluctuation function of order 2 (i.e., $q = 2$) of original and shuffled time series of different geomagnetic storms of the year 2001.

5 Conclusions

In this study, an attempt has been made to study and compare the multifractal behaviour of geomagnetic quiet and disturbed events corresponding to solar cycle 23 using the multifractal detrended fluctuation analysis. Our results show that while the geomagnetic storm events show a conspicuous multifractal behaviour, the solar quiet days show an almost monofractal behaviour. The observed multifractal behaviour of the storm events is determined to be due to the presence of long-range correlations present in the data. Among the storm events considered, the storms corresponding to E-season show similar multifractal behaviour in the morphology of the geomagnetic source, suggesting the presence of some sort of order in the stochastic source morphology. However, more events need to be studied to understand the seasonal dependence of the multifractal behaviour of the source. Our results also clearly demonstrate that the November 06 storm (occurred in relatively less intense winter season) shows a high degree of multifractality in its source, which essentially is due to the prompt penetration of magnetospheric electric fields to low latitudes on November 06, 2001. We believe more data sets of quiet days and disturbed days recorded in different seasons and at different global observatories need to be analysed for a better and comprehensive understanding of the spatio-temporal characteristics of the multifractal behaviour of these extra-terrestrial source field characteristics in a broader perspective.

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