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COMPLEX DEMODULATION AND ELECTROMAGNETIC RESPONSE FUNCTION FOR GEOMAGNETIC FIELD VARIATIONS AT 27-DAY AND ITS HARMONICS

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Abstract

To determine the upper mantle electrical conductivity distribution beyond the depths probed by Sq-periods, the daily mean geomagnetic field variations for the years 1975-1977, recorded through Indo-Russian chain of observatories is subjected to complex demodulation technique to isolate the space-time evolution of periodic field variations for 27-day and its harmonics. The data from this unique latitudinal distribution of 13 observatories has facilitated to best utilize and test statistically the implicit assumption on the representation of spatial characteristic of the field by spherical harmonic term $P_1^0(\cos\theta)$. The isolated demodulates which satisfied the P_1^0 dependence are used to obtain the depth and conductivity of the substitute conductor for period bands of 25-32, 11-16 and 8-10 days, representative of 27-day and its harmonics. These results when combined with those obtained using Sq-periods help to visualize the mantle in the depth range of 50-1200 Km. as a stack of inhomogeneous layers with a clear suggestion of two layers of moderately high conductivity at 125 and 275 Km. and a sharp discontinuity in the depth range of 350-500 Km. The electrical conductivity and the depth of the conductosphere for the period band of 25-32 days are estimated to be 0.33 S/m and 1200 km. respectively. Both these estimates agree quite well with the values reported for southern Chinese and European regions, suggesting a laterally homogeneous electrical structure of the mantle at such great depths.

Introduction

Natural time varying geomagnetic field recorded at the earth's surface is a vector sum of external and internal parts. The external part has its origin in distant current systems in the ionosphere and magnetosphere resulting from the interaction of radiations from the Sun with the earth's permanent magnetic field. The internal part arises due to the currents induced in the earth by external current systems. The ratio of the parts of the magnetic field of internal and external origins is a measure of the electromagnetic (EM) response and is dependent on both the form (spatial) as well as frequency of the external source current system and on the electrical conductivity within the earth. The EM response at different frequencies contain information about the electrical conductivity structure of the earth at different depths. From the skin-depth relationship, the estimation of EM responses at long period geomagnetic variations, eg., solar quiet-day (Sq) variations, magnetic storms and quasi-periodic variations of 27-day and its harmonics, has always been fascinating - for, it provides a means to determine the electrical conductivity distribution at upper mantle depths. The sensitivity of the electrical conductivity to chemical composition, temperature, etc., has been useful for defining the depths of phase transition from Olivine to Spinel as well as to deduce the temperature distribution at upper mantle depths (Akimoto and Fujisawa, 1965; Banks, 1969; Omura, 1991).

Earlier investigations were aimed at obtaining

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a single "global" response function defining a radially symmetric conductivity structure (Lahiri and Price, 1939; Eckhardt et al., 1963; Banks, 1972). However, the growing distribution of geomagnetic observatories coupled with the parallel developments in processing and modeling techniques have led to regional electrical conductivity models. These models reveal large lateral inhomogeneities of the electrical conductivity, atleast in the upper 400 Km. depth (eg., Roberts, 1984; Campbell and Schiffmacher, 1988).

Since the long period geomagnetic variations (for e.g., 27-day and its harmonics, magnetic storms, etc.) are generated by the modulation of the intensity of ring current (Banks, 1969), their spatial structure can adequately be represented by the zonal spherical harmonic $P_1^0(\cos\theta)$ term. This assumption has greatly facilitated the application of 27-day variations to study the deep earth electrical conductivity structure on regional or even for local sites. As a part of the International Magnetospheric Study (1975-77) project, the network of magnetic observatories was augmented along the 143^0 E geomagnetic meridian, providing the best chain of observatories extending from dip-equator at the southern tip of India to the northern parts of Russia and Siberia. In the present study, the daily mean values of horizontal (H) and vertical (Z) field components for the period July 15, 1975 to December 31, 1977 from this chain of stations are subjected to complex demodulation technique to isolate the periodic signals corresponding to 27-day and its sub-harmonics. Unique latitudinal distribution of stations is effectively utilized to test statistically the physical validity of the P_1^0 dependence of spatial behavior of demodulates at selected frequencies. Demodulates which satisfied the P_1^0 dependence are further employed to evaluate the Schmucker's C-response functions, which provided the parameters for a two-layered model, a poorly conducting outer layer overlying a highly conducting inner region, whose EM response is compatible with the observed response at the frequency in question. Such a model is commonly referred to as perfect substitute conductor or conductosphere model (Schmucker, 1970). Determination of the depth of the conductosphere at different long periods serves as a guide to depict the distribution of conductivity at upper and mid-mantle depths. The present study is a sequel to the recently concluded studies, wherein, the Sq variations from the same chain of stations had been utilized to infer the electrical conductivity distribution upto a depth of 500 Km. (Arora et al., 1995) and the analysis of geomagnetic storm-time

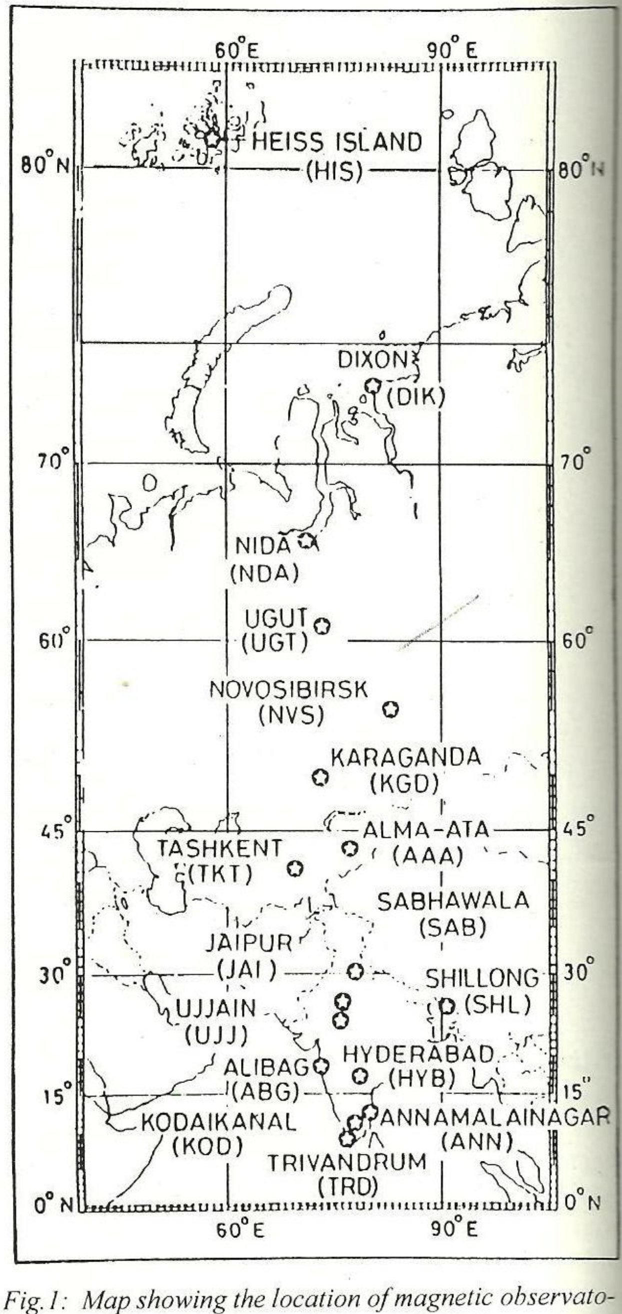

Fig.1: Map showing the location of magnetic observatories along the Indo-Russian sector, whose data are used in the present analysis.

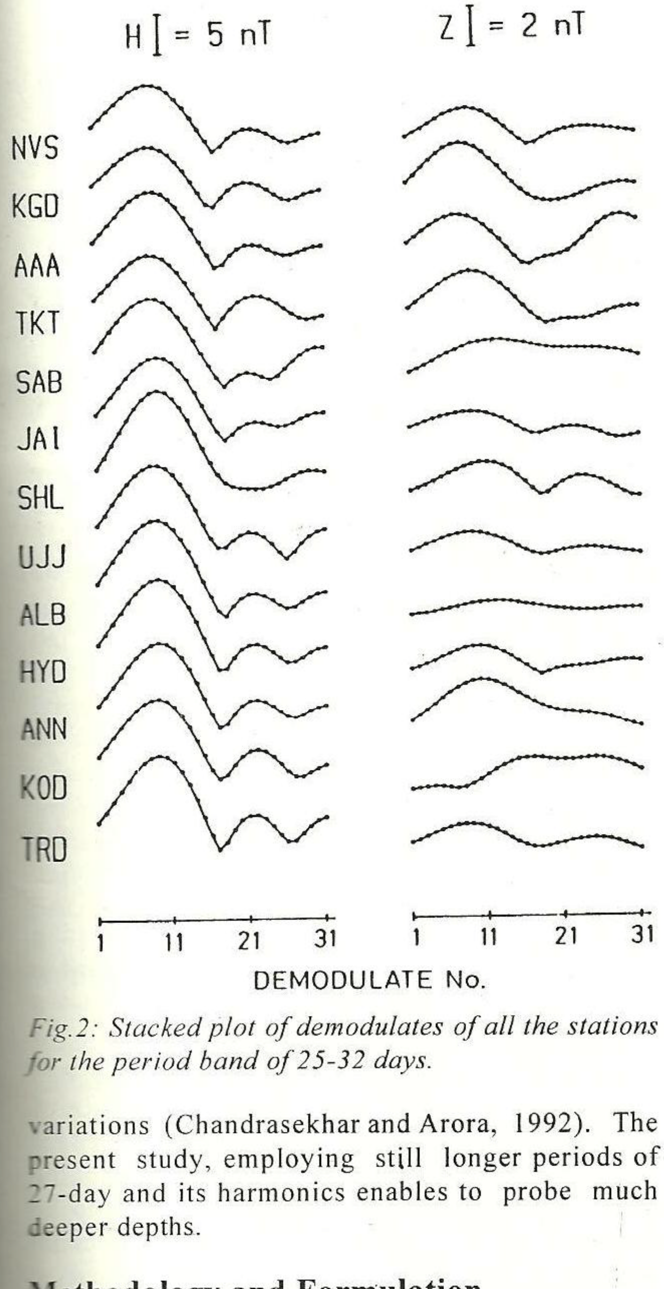

Fig.2: Stacked plot of demodulates of all the stations for the period band of 25-32 days.

variations (Chandrasekhar and Arora, 1992). The present study, employing still longer periods of 27-day and its harmonics enables to probe much deeper depths.

Methodology and Formulation

Data source and time series analysis

Daily mean values of magnetic field components recorded during 1975-77 from 13 observatories shown in fig.1 have been procured to yield 901 data points for each element and for each station. The raw time series is subjected to a 30-point lowpass filtering (Behannon and Ness, 1966) with a cut-off period of 4 days. Then the filtered 841 data points (after losing 30 points on either side of the raw data series) are zero padded to 1728 $(= 2^6 \times 3^3)$ and subjected to a special FFT algorithm, where the total number of data points for computation of FFT can be in the form of $2^m \times 3^n$, where m and n are integers. The FFT spectrum of the filtered series for H-component has shown distinct peaks in the period bands 25-32, 11-16 and 8-10 days at all the stations.

Complex demodulates for selected frequency bands

The spectral estimates corresponding to these three period bands are next subjected to complex demodulation technique outlined in Banks (1975) to obtain the evolution of amplitude and phase of the waveforms with time. This has an advantage over the conventional spectral estimates which are an integrated effect of a frequency component over the whole length of data sequence. In case of complex demodulation technique, each computed demodulate, which is an estimate of the amplitude and phase of the periodic signal at any given segment of time sequence can entirely be treated as an independent entity for testing the spatial behavior as well as calculating the EM response at the given frequency. In the present computation, different period bands viz., 25-32, 11-16 and 8-10 days, centered at 29, 13 and 9 day periods are selected to obtain the complex demodulates both for H and Z time sequences. Towards the computation of complex demodulates, the real and imaginary parts of the Fourier transform are filtered with a suitable discrete function of frequency centered around the peak of interest (say ω) in the chosen frequency band. The filtered sequence is then shifted to zero frequency to have both positive and negative frequency components. The negative half of the frequency component was shifted to positive side and vice versa, so as to avoid any overlap (aliasing) between the two halves. Finally, the new sequence thus obtained was subjected to inverse FFT to convert the transforms back into the time domain to obtain a sequence of complex demodulates centered on the frequency (ω_N) with a sampling interval of $1/2\omega_N$. This procedure has resulted in yielding 31 demodulates in each band for each element and for each station. fig. 2. shows the stacked plot of the amplitudes of 31 demodulates of H and Z at each station corresponding to 25-32 day period band.

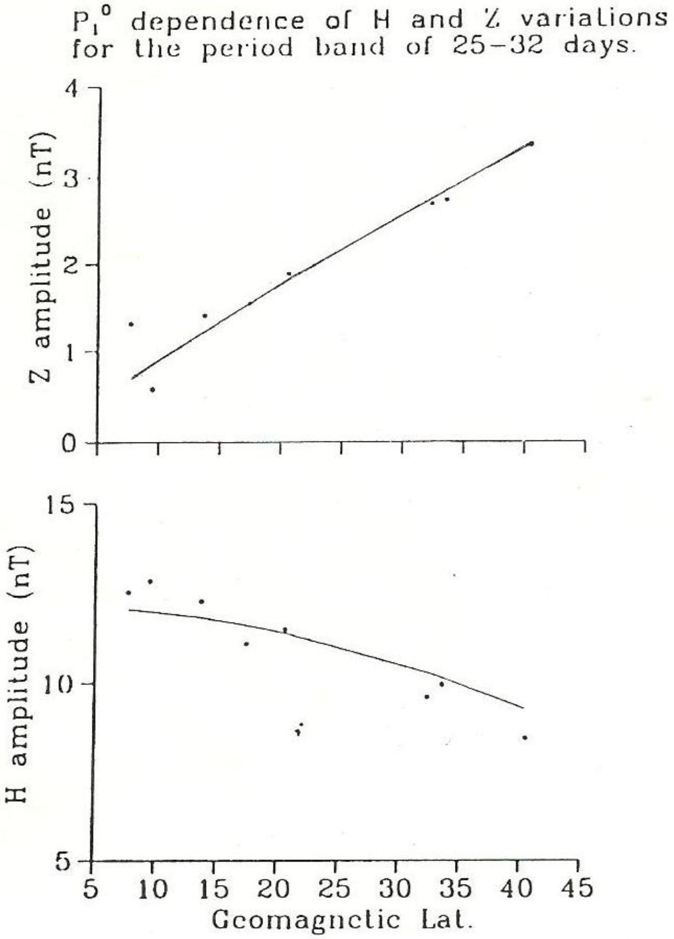

Fig.3: Example plot of latitudinal dependence of H and Z variations for a selected demodulate in the period band of 25-32 days. The solid curve represents the best P_1^0 fitting obtained by iteratively eliminating the stations' data which lay outside the one standard deviation enevelope of least square fitting.

Testing for $P_1^0(\cos\theta)$ dependence

As noted earlier, the 27-day geomagnetic variations are caused by extra terrestrial ring current encircling the earth in equatorial plane at a distance of a few earth radii. Its spatial characteristics at the surface of the earth can be expanded in series of odd order zonal harmonics (Banks, 1969).

$$H(\theta,f) = [\, e_n(f) + i_n(f)] \frac{\partial}{\partial\theta} P_n^0(\cos\theta)$$

$$Z(\theta,f) = [n\, e_n(f) - (n+1)\, i_n(f)]\, P_n^0(\cos\theta)$$

Where H and Z respectively denote the magnetic horizontal (north ward) and vertical (down ward) components respectively. θ is the geomagnetic co-latitude, f denotes the frequency ($= \omega/2\pi$) and n denotes the order of the zonal harmonic. $e_n(f)$ and $i_n(f)$ respectively represent the external and internal parts of the variation field. It follows that, given the origin of long period variations in the ring current, both H and Z variations are expected to exhibit the spatial characteristics which are the functions of $P_n^0(\cos\theta)$. Often due to the lack of spatial distribution of stations along the circle of longitude, the validity of P_1^0 dependence of the inducing source is rather tacitly assumed, than being tested with the data. However, in the present study, since we had data from the unique chain of observatories, we effectively have exploited the possiblilty to show the validity of the P_1^0 dependence of the inducing source for each demodulate of H and Z. fig. 3 shows the typical example of latitudinal variation of amplitude for a selected H and Z demodulate. For reasons of expected contamination of data at high latitudes in association with the auroral electrojet, the data from the stations beyond 60^0 N geographic latitude is not considered for the present analysis. For similar reasons of contamination of data at equatorial regions, because of the influence of equatorial electrojet, due to which the inducing source field is highly non-uniform (Chandrasekhar and Arora, 1994) and due to high anomalous internal field (Singh et al., 1982), the data from the equatorial stations, TRD, KOD and ANN is also not considered for the present study. Least-squares estimation of spatial variation by odd order zonal harmonics (i.e.,P_1^0, P_3^0, etc.) showed that the latitudinal dependence of the inducing source field can largely be accounted for by the single P_1^0 term, as the contribution from the other odd order zonal harmonics are found to be negligibly small. Hence P_3^0 and higher harmonics are not considered in the present analysis. The solid curves in fig.3 represents the best P_1^0 fitting obtained by iteratively eliminating the stations' values which lay outside the one standard deviation enevelope of least-squares fitting. The electromagnetic response is computed only for those demodulates which strictly satisfied the P_1^0 criterion.

Computation of Electromagnetic Response

The electromagnetic response of the earth for odd order zonal spherical harmonics at a given frequency, as defined by Banks (1969) is given by,

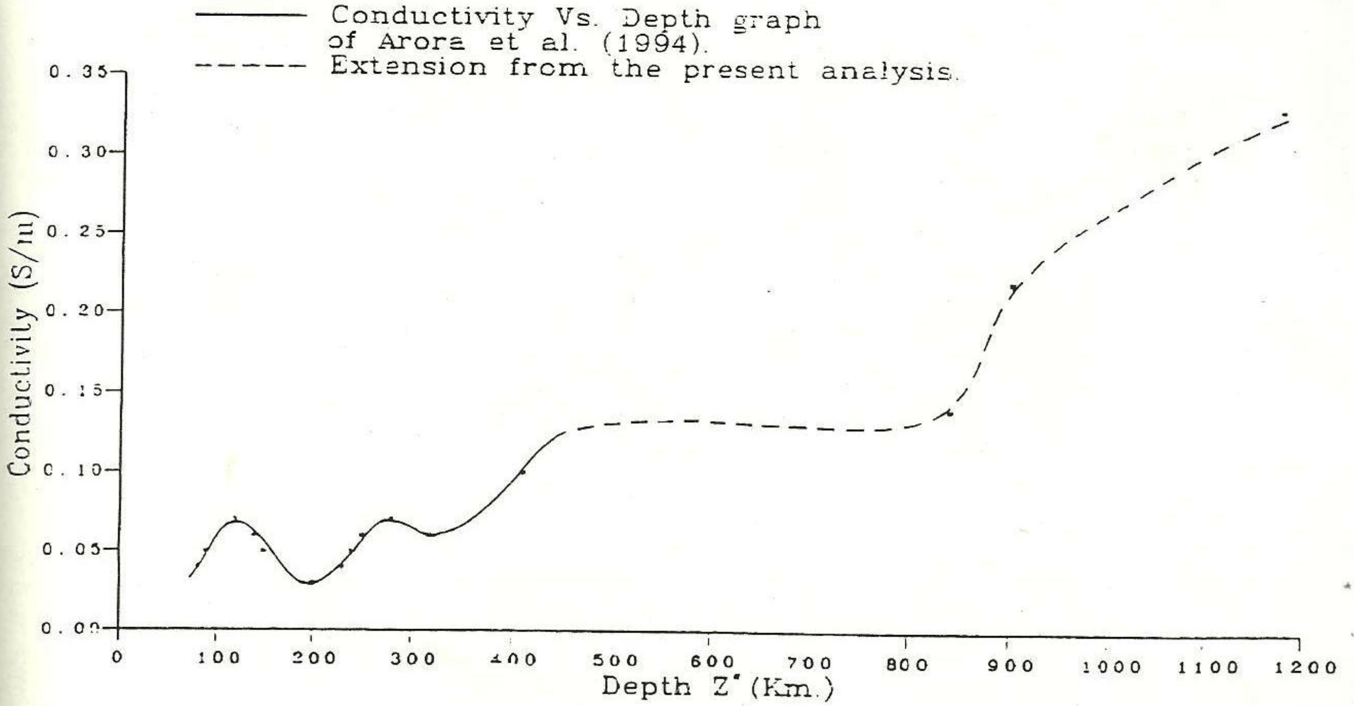

Fig.4: Plot of conductivity Vs. depth of the conductosphere for the periods of Sq and 27-day and its harmonics.

$$W_n(f) = \frac{n e_n(f) - (n+1)\, i_n(f)}{e_n(f) + i_n(f)} = -\tan\theta \left[\frac{Z(\theta, f)}{H(\theta, f)}\right]$$

Since P_3^0 and higher order harmonics are neglected due to their negligibly low amplitudes compared to P_1^0 amplitude, only W_1 response is calculated. According to the perfect substitute conductor model of Schmucker (1970), the inductive scale length C is given by

$$C = R \frac{(1 - 2Q_1)}{2(1 + Q_1)}$$

where $Q_1 = (1 - W_1) / (2 + W_1)$

and R is the Radius of the earth (6371 Km.).

Here, the real part of C gives the depth (Z*) of the conductosphere, where the induced currents are highly concentrated and the imaginary part of C (C_{imag}) is used to calculate the conductivity of the conductosphere by using the formula (Chen and Fung, 1991)

$$\sigma = 0.253 \times 86400 \times T / (4 \times (C_{imag})^2)$$

Where T is the period of the inducing field in unit of days. The mean W_1 - response and the mean conductosphere depth (Z*), (i.e., A_R) together with their standard errors (δ_R) for each band is calculated by the weighted average method of Beamish and Banks (1983) by

$$A_R = \sum_{k=1}^{N} a_k \cdot A_{Rk} \Big/ \sum_{k=1}^{N} a_k$$

$$\delta_R^2 = 1 \Big/ \sum_{k=1}^{N} a_k$$

where the weighting function $a_k = 1/(\delta_k)^2$. δ_k being the standard error associated with each individual determination of A_{Rk}. N denotes the total number of those demodulates in each band which strictly satisfied the P_1^0 criterion. The weighted mean conductivity (σ) for each period band was calculated by using the weighted mean of C_{imag} and the central peak period of that respective band.

Results and Discussion

The weighted mean values of W_1, conductosphere depth (Z*) together with their associated error estimates and conductivity (σ) of the conductosphere are give in Table-I for the three period bands viz., 25-32, 11-16 and 8-10 days, representative of 27-day variation and its harmonics. The results clearly show that the depth of the

Table-I

Electromagnetic response estimates(W_1), conductosphere depths and the conductivities of the conductosphere for three different long period bands depicting the 27-day variations and and its harmonics. The values in parantheses depict the corresponding associated error estimates.

Period band (days)	Weighted mean W_1 response	Weighted mean conductosphere depth (Z^*)	Weighted mean conductivity σ(S/m)
25-32	0.43 (± 0.01)	1175(± 20)	0.33
11-16	0.33 (± 0.01)	900 (± 13)	0.22
8-10	0.32 (± 0.01)	840 (± 50)	0.14

Table-II

Comparison of response function estimates for period band of 25-32 days (present study) with those of southern Chinese and global values obtained for 27-day variation and its harmonics.

Estimates	Indo-Russia sector	Southern Chinese region (Chen and Fung, 1991)	Global
Depth (Z^*)(km)	1175(±20)	1000-1300	800
Conductivity(σ) S/m	0.33	0.3-0.4	

conductosphere (depth of the highly conducting core underlain by the poorly conducting layer) increases with the increasing period of geomagnetic field variations. A complete picture of the conductivity-depth profile obtained by combining the present results and those obtained by Arora et al.(1995) from the analysis of Sq variations is shown in fig. 4. It can be clearly seen in this figure, that the mantle upto a depth of about 1200 Km. can be viewed as a stack of inhomogeneous layers. The results of the analysis of Sq variations indicated the conductivity values of about 0.06 S/m from 50 to approximately 350 Km., with relative maxima near 125 and 275 Km, interspersed with relative minima near 270 to 330 Km. Thereafter, the conductivity increases sharply to about 0.18 S/m at 500 Km. depth with no signs of leveling off. The marginal increase in conductivity near the 125 Km. depth was reckoned to mark the conductivity transition between lithosphere and asthenosphere. The slight increase in conductivity around 275 Km. shows a good correlation with the low velocity zone in the regional velocity structure model of the Himalaya (Beghoul et al., 1993) to which the derived conductivity profile is sensitive. The correspondence between high conductivity and low velocity layers coupled with the laboratory results suggests that among the major mantle materials, pyroxene is the most stable repositors of the hydrous phase and this led Arora et al. (1995) to suggest the hydrogen saturated pyroxene as the dominant conductors in the depth range of 275 Km.

At the lowest period (9 days) examined in the present study, the conductosphere depth is estimated to be about 800 Km. The large gaps in the depths probed by Sq (upto 500 Km) and the lowest period (9 days) (≈ 800 Km.) is not able to constrain the exact nature of sharp increase of conductivity around 400 Km. depth, but the near comparable value of conductivity around 500 Km. (provided by Sq periods) and 800 Km. (provided by 9-day period) suggest that the continuously rising values of conductivity from 350 to 500 Km. taper off to a mean value of 0.15-0.2 S/m. in the depth range of 500 - 800 Km. Thereafter, the conductivity shows again a sign of sharp increase from 0.15 S/m to 0.33 S/m at around 1200 Km. depth.

Table-II shows the comparison of response function estimates for the period band of 25-32 days from the present study with the southern Chinese amd global values obtained for 27-day period. It is interesting to note that in the period band of 25-32 days, centered at 29-days, both the estimated depth and conductivity of the conductosphere along the Indo-Russian sector shows a fair compatibility with the results obtained in the southern Chinese region (Chen and Fung, 1991 and European region (Pecova et al., 1980). It is also a worthy of note, that the estimated depth of conductosphere of about 1200 Km. agrees quite well with the global conductosphere depth obtained by Magsat studies (Langel, 1990). This agreement suggests that the electrical conductivity structures beneath the Indo-Russian and Southern Chinese sectors can be considered to be laterally homogeneous at this depth range of the mantle. It is well known that as the depth of penetration of inducing field increases with increasing periods, the larger volume of the earth tend to control the measured EM response. As a consequence of this, the effect of the near surface electrical discontinuities is diluted, thus making the measured response sensitive to deeper conductivity distribution. The close agreement of the measured induction response in the period band of 25-32 days along Indo-Russian, southern China as well as global values strongly support the idea that the electrical conductivity structure of the continents become more and more laterally homogeneous at these great depths (Roberts, 1986).

Conclusions

Unique latitudinal distribution of stations' data employed in the present study has facilitated to test statistically the validity of the implicit assumption of P_1^0 dependence of the inducing source field for long period geomagnetic variations. Coupled with the complex demodulation technique, wherein each demodulate can be treated entirely as an independent entity, this has further facilitated to use only those demodulates which strictly satisfied the P_1^0 criterion, for calculating the electromagnetic response.

It is observed that at the long periods examined, the effect of near surface lateral inhomogeneities are averaged out and the computed EM response functions are sensitive to radial electrical conductivity variations. The computed response functions corresponding to three different period bands viz., 25-32, 11-16 and 8-10 days, which are representatives of 27-day variation and its harmonics suggest a continuous increase in conductivity with increasing depths. The conductivity and depth of the conductosphere for the period of 29 days is estimated to be 0.33 S/m and 1200 Km. respectively. This agrees quite well with the obtained conductivity and depth estimates beneath the southern Chinese region, suggesting a laterally homogeneous electrical structure of the mantle at these large depths along the Indo-Russian sector and the southern Chinese region.

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