J. Ind. Geophys. Union (January 2018)
v.22, no.1, pp: 49-51

Locating Center of Pressure in 2D Geological Situations

Soumyajit Mukherjee
Department of Earth Sciences, Indian Institute of Technology Bombay
Powai, Mumbai 400 076, Maharashtra, INDIA
*Corresponding Author: soumyajitm@gmail.com

ABSTRACT
Center of pressure (COP) for horizontal rock slices with realistic density distribution is presented. The location of the COP within the slab depends on the following parameters: (linear) density gradient, compaction constant, density of matrix and that of the pore fluid, and the length and width of the slab. However, no simple proportionality relation amongst the co-ordinates of the COP and these parameters exist. Vertical and thin rock layers such as sedimentary and igneous dykes with different (empirical) relations of spatial density variation can also be worked out in a similar way to locate their COPs.

Key words: Statics, porosity, density distribution, tectonics, structural geology

INTRODUCTION

The center of pressure (COP) is a well-established concept in statics (e.g., Das and Mukherjee 2012). The term sometimes is also used in geosciences in various contexts. For example, the depth of overpressure in a magma chamber is related with the location of the COP within it (Clarke et al., 2007). Tectonic movement and volcanism have been linked with the relocation of the COP (Nishi et al., 2007; also Kumalasari and Srigutomo 2016). Volume and pressure change in a magma reservoir can relocate the COP leading to seismicity (Decker et al., 1983). Exploration for hydrocarbons sometimes, requires drilling to be made close to or at the center of pressure of the reservoir (Chin 2016). However, no direct connection between COP and these geological factors have been explored.

This work makes a preliminary analysis of COP location for a laminar rock slice with geologically realistic density distribution downward. COP for a 3D (irregular) object appears not to be straightforward to deduce (e.g., Bunimovick and Dubinskii 1982) and could be picked up as the follow up detailed work.

THE GEOLOGICAL MODEL

Background

Consider a rectangular lamina/slice of rock immersed in a fluid. We choose X-axis horizontal and the Y-axis vertical, as in Figure 1. In this case the COP will have coordinates (Das and Mukherjee 2012):

$$x=\int_0^{x_1}\int_0^{y_1}\rho(x,y)\,x\,dx\,dy \Big/ \int_0^{x_1}\int_0^{y_1}\rho(x,y)\,dx\,dy \quad \text{(i)}$$

And,

$$y=\int_0^{x_1}\int_0^{y_1}\rho(x,y)\,y\,dx\,dy \Big/ \int_0^{x_1}\int_0^{y_1}\rho(x,y)\,dx\,dy \quad \text{(ii)}$$

Case 1:

Consider a rectangular parallelepiped and co-ordinate axes as shown in the Figure 1. Density at the origin (0,0) be ρ_0, and the linear density gradient in perpendicular directions are k_i (i=x,y). Mukherjee (2017) reviewed geological cases of crust, lithosphere and sedimentary basins where such density gradients have been reported. Therefore, density variation along X-, Y- and Z-axes are:

$$\rho(x,0) = \rho_0+k_x x \quad \text{(iii)}$$

$$\rho(0,y) = \rho_0+k_y y \quad \text{(iv)}$$

Therefore, for any coordinate (x,y), the density would be given by

$$\rho(x,y) = \rho_0+k_x x+k_y y \quad \text{(v)}$$

Note, putting x=0 and y=0 in two separate cases, one can go back to eqns (iii) and (iv), respectively.

Putting the expression of ρ(x,y) into eqns (i) and (ii) and performing the definite integral in the numerator and the denominator, the COP coordinates are:

$$x = x_1\{0.5*\rho_0+0.33*k_x.x_1+0.25*k_y y_1\}\ \{\rho_0+0.5*(k_x x_1+k_y y_1)\}^{-1} \quad \text{(vi)}$$

$$y = y_1\{0.5*\rho_0+0.25*k_x.x_1+0.33*k_y y_1\}\ \{\rho_0+0.5*(k_x x_1+k_y y_1)\}^{-1} \quad \text{(vii)}$$

Note for a homogeneous slab with $k_x= k_y=0$, the coordinate simplifies to ($0.5*x_1$, $0.5*y_1$), which is the centroid of the slab.

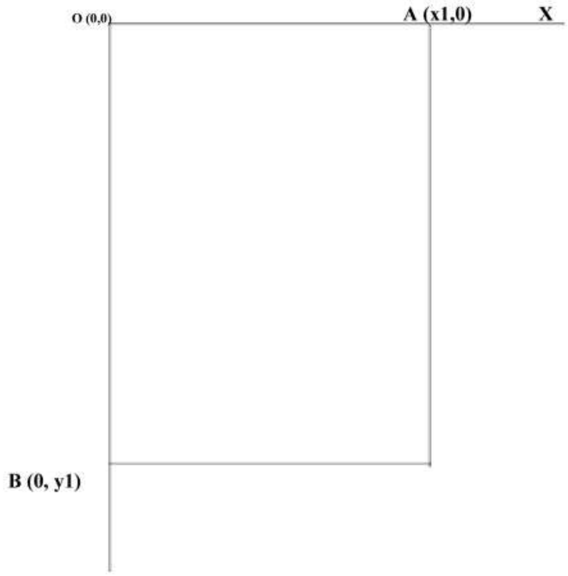

Figure 1. The rectangular thin slab of rock along with the chosen co-ordinate axes.

Case 2:

The vertical density variation can vary significantly from linearity , and can be represented as (Mukherjee, 2017):

$$\rho_{bwy} = \rho_m - (\rho_m - \rho_f)\phi_0 e^{-by} \tag{viii}$$

where

ρ_{bwy}: Bulk wet density of sediment at depth 'y'; ρ_m: density of matrix; ρ_f: density of pore fluid; ϕ_0: surface porosity; b^{-1}: compaction constant.

Considering a linear variation of density along X-direction as in eqn (iii),

$$\rho(x,y) = k_x x + \rho_m - (\rho_m - \rho_f)\phi_0 e^{-by} \tag{ix}$$

For x=y=0, i.e. at origin, the density of $[\rho_m - (\rho_m - \rho_f)\phi_0]$ is considered.

Putting the expression of $\rho(x,y)$ into eqns (i) and (ii) and performing the definite integral in the numerator and the denominator, the COP coordinates are:

$$x = A\,B^{-1} \tag{x}$$

$$y = C\,B^{-1} \tag{xi}$$

Here

$$A = x_1^2[0.5*\rho_m y_1 - 0.5*b^{-1}(\rho_m - \rho_f)\,\phi_0(1 - e^{-by1}) + 0.33\,k_x x_1 y_1] \tag{xii}$$

$$B = x_1[0.5*\rho_m y_1^2 + (\rho_m - \rho_f)\,\phi_0 b^{-1} e^{-by1}(y + b^{-1}) + 0.25 k_x x_1 y_1^2] \tag{xiii}$$

$$C = x_1 y_1\{\rho_m + 0.5*k_x x_1\} + (\rho_m - \rho_f)\,\phi_0 x_1 b^{-1}(e^{-by1} - 1) \tag{xiv}$$

Note, here for k_x=b=0, i.e., for a homogeneous slab with density ρ_m, x=0.5*x_1, and y=0.5*y_1. This point is the center of gravity, which matches with our intuition.

DISCUSSIONS

Case 1 works for igneous intrusions, and case 2 especially for sedimentary slabs/dykes. In case density variation is present just along one direction, COP can still be deduced by taking one of the k_i = 0 (case 1), or by taking either k_i = 0 or b = 0 (case 2). The coordinate depends on the length and the width of the slice (x_1 and y_1), the density gradient (k_x, k_y), compaction constant (b^{-1}), surface porosity (ϕ_0), matrix density (ρ_m) and the fluid density (ρ_f). However no simple proportionality relation exists between the co-ordinate and any of these parameters. The present analysis (eqns i to xiv) holds true for very thin vertical igneous and sedimentary dykes, which do exist in nature (Alm and Sundbond 2002). The deductions will become approximate if the dip of the dyke is not perfectly vertical, or if the dyke is of several meters width (Fodor and Kazmer 1989). Thinner the dyke and steeper it dips, closer will be the match with the presented COP analysis. Vertical basalt dykes are common (Wellman and Wilson 1964) and mm-scale thin varieties are rare (Krumbholz et al., 2014). Sedimentary dykes passively fill up the space created by tectonic fractures (Roshoff and Cosgrove 2002). The present analysis works if the tectonic stresses cease and hydrostatic condition prevail in the sub-surface.

A three dimensional extension of the concept COP in terms of its coordinate (x,y,z) can be made by writing:

$$x = \int_0^{x_1}\int_0^{y_1}\int_0^{z_1} \rho(x,y,z)\, x\, dx\, dy\, dz \Big/ \int_0^{x_1}\int_0^{y_1}\int_0^{z_1} \rho(x,y,z)\, dx\, dy\, dz \tag{xv}$$

$$y = \int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z)\, y\, dx\, dy\, dz \Big/ \int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z)\, dx\, dy\, dz \qquad \text{(xvi)}$$

$$z = \int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z)\, z\, dx\, dy\, dz \Big/ \int_0^{x_1} \int_0^{y_1} \int_0^{z_1} \rho(x,y,z)\, dx\, dy\, dz \qquad \text{(xvii)}$$

This is the same as the center of gravity of the rectangular parallelepiped with length z_1 along the Z-axis. Note that the densities of compacting sediments and cooling igneous materials will naturally be time dependent. Therefore, the presented model requires improvements to deal with such events over a relatively longer geological time period.

ACKNOWLEDGEMENTS

A research sabbatical for the year 2017 and a research grant provided by IIT Bombay to SM are thanked. Positive review by Prof. B.V.S. Murty and expert handling of editing by him and chief editor are acknowledged. "Vide Mukherjee (2017, submitted-1,2) for issues on isostasy, center of gravity and moment (/product) of inertia for an inhomogeneous crustal layer as has been considered here."

Compliance with Ethical Standards

The author declares that he has no conflict of interest and adheres to copyright norms.

REFERENCES

Alm, E., and Sundbland, K., 2002. Fluorite-calcite-galena-bearing fractures in the counties of Kalmar and Blekinge, Sweden. Svensk Kärnbränslehantering AB. ISSN 1402-3091.

Bunimovick, A.I., and Dubinskii, A.D., 1982. On the center of pressure of a body. Fluid Dynamics 17, pp: 760-764.

Chin, W.C., 2016. Reservoir Engineering in Modern Oilfield: Vertical, deviated, horizontal and Multi-layered well systems. Handbook of Petroleum Engineering Series.Wiley. v.1, pp: 135.

Clarke, A.B., Stephens, S., Teasdale, R., Sparks, R.S.J., and Diller, K. 2007. Petrologic constraints on the decompression history of magma prior to Vulcanian explosions at the Sourfriere Hill volcano, Montserat. J. Volcanol. Geotherm. Res. v.161, pp: 261-274.

Das, B.C., and Mukherjee, B.C., 2012. Integral Calculus: Differential Equations. U.N. Dhur & Sons Private Limited. 55th Edition. pp: 554.

Decker, R.W., Koyanagi, R.Y., Dvorak, J.J., Lockwood, J.P., Okamura. A.T., Yamashita, K.M. and Tanigawa, W.R. 1983. Seismicity and surface deformation of Manua Loa Volcano, Hawaii. EOS v.64, no.37.

Fodor, L., and Kázmér, M., 1989. Clastic and carbonate sedimentation in an Eocene strike-slip basin at Budapest. Field Guidebook. pp: 227-259.

Krumbholz, M., Hieronymus, C.F., Burchardt, S., Troll, V.R., Tanner, D.C., and Friese, N. 2014. Weibull-distributed dyke thickness reflects probabilistic character of host-rock strength. Nature Communications 5. DOI: 10.1038/ncomms4272

Kumalasari, R., and Srigutomo, W., 2016. Location and pressure change prediction of Borneo volcano magma chamber using inversion scheme. J. Physics: Conf. Series: 739, 6th Asian Pacific Symposium.

Mukherjee, S., 2017. Airy's isostatic model: a proposal for a realistic case. Arabian Journal of Geosciences 10:268.

Mukherjee S. (Submitted-1) Locating center of gravity in geological contexts. International Journal of Earth Sciences.

Mukherjee S. (Submitted-2) Moment of inertia for rock blocks subject to bookshelf faulting with geologically plausible density distributions. Journal of Earth System Science.

Nishi, K., Hendrasto, M., Mulyana, I., Rosadi, I. and Purbawinata, M.A. 2007. Micro-till changes preceding summit explosions at Semeru volcano, Indonesia. Earth Planets Space v.59, pp: 151-156.

Roshoff, K., and Cosgrove, J., 2002. Sedimentary dykes in the Oskarshamn-Västervik area :A study of the mechanism of formation. Svensk Kärnbränslehantering AB. ISSN 1402-3091. pp: 1-98.

Wellman, H.W., and Wilson, A.T., 1964. Notes on the geology and archaeology of the Martins Bay district. New Zealand J. Geol. Geophys. v.7, pp: 702-721.

Received on: 25.7.2017; Revised on: 21.8.17; Re revised on: 13.9.17; Accepted on: 29.9.17.