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Kinematics of "top –to-down" simple shear in a Newtonian Rheology

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INTRODUCTION

'Simple shear' is a homogeneous deformation, where a line bound by two parallel boundaries deforms into another straight line by shear of one or both the boundaries (Twiss and Moores 2007) (Fig. 1a). Frehner et al. (2011) pointed out problems in analogue models (to generate homogeneous deformation of such simple shears) and the analytical precautions that could minimize inhomogeneity. A simple shear, in which the top boundary shears upward relative to the bottom one is designated as 'top-to-upward' shear, whereas a downward movement ~~as~~ a 'top-to-downward' shear.

MODEL

The model considers a shear zone of a single lithology of an incompressible Newtonian rheology bound by two very long parallel dipping boundaries. A 'top-to-down-dip' shear of the boundaries is considered under a pressure gradient that tends to extrude the rock. A component of gravity counteracts with this. The kinematics of such a shear zone is deduced based on the Poisson equation of flow (eqn 1). Thermal effects of the shear zone is neglected following Jain et al. (2005), Mukherjee (2007, 2011), Mukherjee and Koyi (2010 a,b), Mukherjee et al. (2012), Frehner et al. (2011), Mukherjee et al. (2011), Mukherjee and Mulchrone (2012), Mukherjee (2013 a,b), Koyi et al. (2013).

The 'Poisson equation' flow of an incompressible Newtonian fluid in the z-direction through rigid boundary (inclined shear zone) is (Papanastasiou, et al 2000):

$$(\partial^2 U_z/\partial x^2) + (\partial^2 U_z/\partial y^2) = \mu^{-1} [\partial P/\partial z - d\ g\ \mathrm{Sin}\theta] \quad (1)$$

'x' and 'y': are perpendicular directions that lie on the cross-section of the shear zone; U_z- fluid along z-direction; 'μ'- fluid viscosity; $(\partial P/\partial x)$- pressure gradient leading to extrusion; 'd': fluid density; 'g': gravitational acceleration; and 'θ': shear zone dip.

Considering only the YZ section, $(\partial^2 U_z/\partial x^2) = 0$. Therefore:

$$(\partial^2 U_z/\partial y^2) = \mu^{-1} [\partial P/\partial z - d\ g\ \mathrm{Sin}\theta] \quad (2)$$

Integrating twice, considering the shear zone to be of $2y_0$ units thick, and at $y = y_0$, $U_z = -U_1$, and at $y = -y_0$, $U_z = U_2$ one can deduce the profile:

$$U_z = 0.5\ \mu^{-1} [\partial P/\partial z - d\ g\ \mathrm{Sin}\theta] (y^2 - y_0^2) + 0.5 \{(U_2 - U_1) - y\ y_0^{-1} (U_1 + U_2) \} \quad (3)$$

When $d\ g\ \mathrm{Sin}\theta = \partial P/\partial z$, eqn (3) simplifies to:

$$U_z = 0.5 \{(U_2 - U_1) - y\ y_0^{-1} (U_1 + U_2) \} \quad (4)$$

The 'y' ordinate of the vertex of the profile in eqn (3) is given by:

$$y_1 = 0.5\ y_0^{-1} \mu (U_1 + U_2) (\partial P/\partial z - d\ g\ \mathrm{Sin}\theta)^{-1} \quad (5)$$

For $y = 0.5\ y_0^{-1} \mu (U_1 + U_2) (\partial P/\partial z - d\ g\ \mathrm{Sin}\theta)^{-1}$ (6)

the vertex of the profile touches the upper boundary of the zone.

For $y < 0.5\ y_0^{-1} \mu (U_1 + U_2) (\partial P/\partial z - d\ g\ \mathrm{Sin}\theta)^{-1}$ (7)

The vertex lies outside the shear zone.

When $d\ g\ \mathrm{Sin}\theta = \partial P/\partial z$; and $U_1 < U_2 < 0$, the profile becomes (Fig. 2e):

$$U_z = 0.5 \{(U_1 - U_2) + y\ y_0^{-1} (U_1 + U_2)\} \quad (8)$$

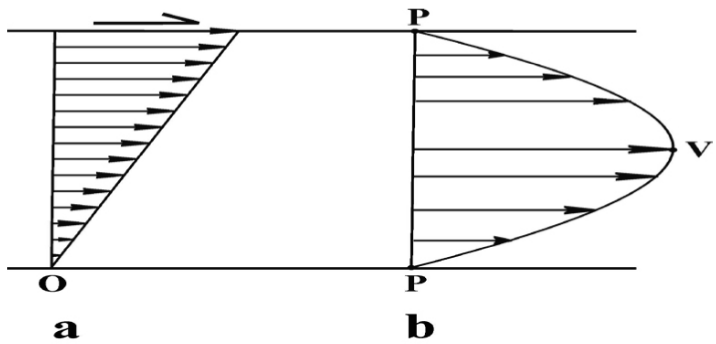

Figure 1. a. Simple shear: linear profile; b. Poiseuille flow of Newtonian fluid, parabolic profile. Full arrows: flow direction.

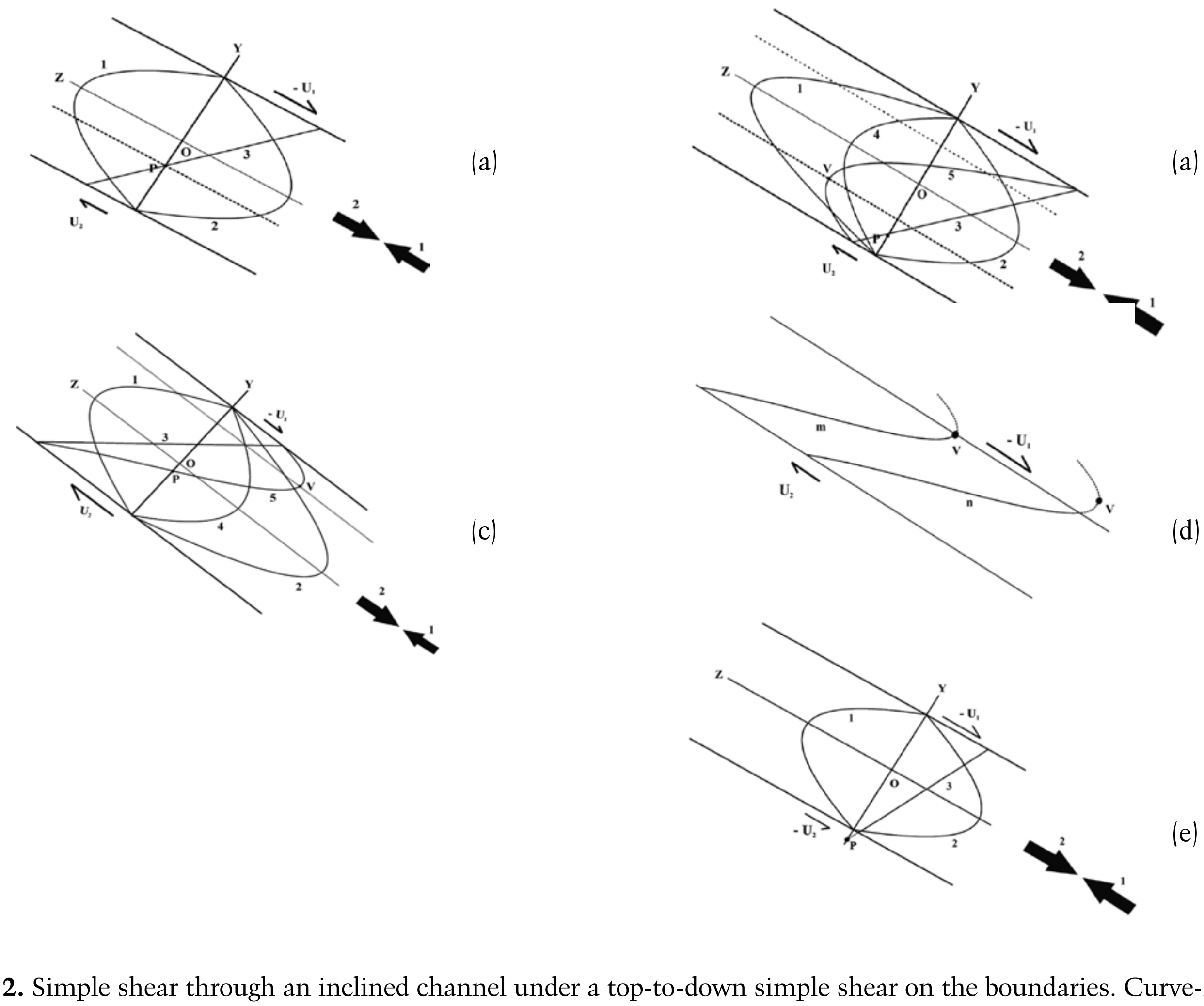

Figure 2. Simple shear through an inclined channel under a top-to-down simple shear on the boundaries. Curve-2: flow due to gravity; Curve-1: flow by pressure gradient; Curve 4: combination of flows '2' and '1'; Curve-3: flow due to shear of boundaries; Curve-5: the resultant flow profile. Full arrows '1' and '2': intensities of the respective flows; V: Vertex; P: Pivot. Dash line: demarcate sub-zones of opposite shear senses. Cases: a. Gravity component = pressure gradient. b. Gravity component < pressure gradient. c. Gravity component > pressure gradient. d. Gravity component > pressure gradient along with specific algebraic relation amongst geometric and rheologic parameter of the shear zone, velocity of shear at the boundaries and the pressure gradient (eqns 6 and 7 in the Appendix). e. gravity component = pressure gradient and both the boundaries shear downward with the top one at a higher velocity.

CONCLUSIONS

The following observations can be made:

When the gravity component of flow equals the pressure gradient, and boundaries shear, a linear profile forms (Fig. 2a, compare with Fig.1a; eqn 4). A uniform shear sense develops. The point of intersection between the profile and the the Y-axis defines the static point *'pivot'*. Across a line passing through the pivot that parallels the boundary the direction of movement of points in the shear zone is opposite. While the upper sub-zone of the inclined shear zone moves down, the bottom sub-zone moves upwards.

When the gravity component of flow is weaker than the pressure gradient and the boundaries shear, the resultant parabolic profile tapers towards the up-dip side with its *vertex* lying inside the lower half of the master shear zone (Fig. 2b; eqn 3). Across a line passing through the vertex that parallels the shear zone boundary demarcates two sub-zones of reverse shear sense. While the top-sub-zone shows a shear sense same as that applied on the boundaries, the bottom thinner sub-zone shows the reverse.

If the gravity component of flow is stronger than the pressure gradient while the boundaries shear, the resultant parabolic profile tapers downdip but its *vertex* lies inside the upper half of the master zone (Fig. 2c; eqn 3). In this case, the top sub-zone produces a shear reverse to that at boundaries, whereas the thicker sub-zone at its bottom shows the same sense.

When parabolic profile is the outcome of a simple shear, the vertex may lie on one of the boundaries of the inclined shear zone (curve 'm' in Fig. 2d), or even outside (curve 'n' in Fig. 2d). These could happen under special relation amongst the thickness and dip of the shear zone, density and viscosity of the rock mass, and shear velocity on the boundaries, and the pressure gradient that tends to extrude the rock (eqns 6 and 7). A uniform shear forms.

When both the boundaries shear downwards, but the upper one with a higher velocity, the pivot lies outside the shear zone. A special case of pressure gradient equal to the gravity component is shown in Fig. 2e (eq. 8). Here also, a uniform shear sense develops.

An analytical model of simple shear is presented for a shear zone with parallel boundaries under a 'top-to-down-dip' shear on an incompressible Newtonian fluid. A pressure gradient that tends to extrusion, counteracts with the effect of gravity that leads to flow down-dip. A linear profile forms only when the gravity and the pressure gradient components balance each other resulting in a uniform shear sense. If those two components are unequal and the boundaries shear, a parabolic profile forms where the vertex delineates the boundary between two sub-zones of reverse shear. The pivot is the intersection between the velocity profile and line with respect to which the profile is constrained. The pivot remains static. Across a line passing through the pivot that parallels the boundary divides the master shear zone into an upper- and a lower sub-zone. A single shear sense develops inside the shear zone if under specific relation amongst the flow parameters the vertex of the flow profile touches one of the boundaries, or lie outside the master shear zone. The pivot lies outside the shear zone if both the boundaries move in the same direction and gravity component equals pressure gradient. To what extent the model holds true if viscous dissipation (Mukherjee and Mulchrone, 2013) needs to be studied.

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