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Shear heating by translational brittle reverse faulting along a single, sharp and straight fault plane

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Shear heating by reverse faulting on a sharp straight fault plane is modelled. Increase in temperature (T_i) of faulted hangingwall and footwall blocks by frictional/shear heating for planar rough reverse faults is proportional to the coefficient of friction (μ), density and thickness of the hangingwall block (ρ). T_i increases as movement progresses with time. Thermal conductivity (K_i) and thermal diffusivity (k_i') of faulted blocks govern T_i but they do not bear simple relation. T_i is significant only near the fault plane. If the lithology is dry and faulting brings adjacent hangingwall and footwall blocks of the same lithology in contact, those blocks undergo the same rate of increase in shear heating per unit area per unit time.

1. Introduction

Heat produced due to conversion of mechanical work in brittle deformation regime by sliding one rock unit over the other along fault planes is called 'frictional heating'/'shear heating'. Study of shear heating is important in earthquake- (Fulton and Harris 2012) and landslide-studies (Goren and Aharonov 2007), contact metamorphism (Graham and England 1976), and thermal structures of fault zones (Lamb 2006). Such studies are also important to understand the genesis of pseudotachylites (Vernon and Clarke 2008) that may be present along faults (Rice and Cocco 2005), fault kinematics (e.g., Sibson 2002), thermal softening during deformation (Blanpied *et al.* 1998), etc. (also see Segall and Rice 2006). Shear heating can be significant since a fast moving fault ($\geq$1 cm yr^{-1}; Scholz 1990) can elevate the temperature to $> 250°$C within tens of seconds (Kitamura *et al.* 2012), but that is restricted to the vicinity of the fault plane (Wibberley *et al.* 2008). Despite several researches on shear heating by faulting under a number of boundary conditions, a model based on the minimum and the most critical parameters seems to be lacking.

Even though doubted (Cardwell *et al.* 1978), shear heating along faults have been proved from various evidences (reviewed in Nemčock *et al.* 2005). The manifestation of shear heating is ubiquitous when faults with fast slip rates cut across cold and tectonically stable crustal parts devoid of pore fluids (Sibson 2002), and temperatures have been documented to reach as high as ∼1080°C (Hamada *et al.* 2009). In some cases, however, evidences of high shear heating were obscured, possibly by the flow of groundwater (Artemieva 2011). The effect of shear heat is decipherable in terms of thermal anomaly or elevation of temperature (such as the thermal aureoles described by Annen *et al.* 2006) and/or when shear stress exceeds 15–20 MPa (Chester *et al.* 1993). Thrust fault related shear heating has been proven to be significant at the beginning of deformation (Graham and England 1976). Slip rate and rock strength are the two important parameters for shear heat (Graham and England 1976). The assumption of previous workers that (almost) all work done by faulting converts

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to heat energy (e.g., Lockner and Okubo 1983) is followed here. The presented deductions are in fact much simpler than many available advanced models on shear heat (such as Kitajima *et al.* 2010). However, the following constraints are not considered in this work: shear heating due to splay faulting and fracturing (Devès *et al.* 2014), localisation of shear heat along a fault due to difference in asperity, thermal softening of the fault, variation of slip rate along the fault, development of gouge and/or breccia aiding modification of the frictional coefficient, the role of pore fluids and secondary mineralisation along the fault plane, effect of geothermal gradient (Passelegue *et al.* 2014), logarithmic relation between frictional coefficient and rate of slip (Noda 2008), dilation of fault zone (Garagash and Rudnicki 2003b), temporal fall in frictional coefficient as faulting happens (Rice and Cocco 2005), change in asperity of the fault surface (Beeler *et al.* 2008), etc.

During brittle shearing, some of the parameters, however, remain unchanged such as the frictional coefficient for rocks (μ) at high confining pressure (Davis and Reynolds 2012), though cases of changing μ exist (Middleton and Wilcock 1994). Planar inclined brittle reverse shear planes devoid of gouge, breccia and damage zones do exist in nature (e.g., figure 6a of Mukherjee 2013a; also see Mukherjee 2007, 2010a, b, 2013b, 2014, 2015; Mukherjee and Koyi 2010a, b; etc.). The deductions apply there. This work presents a simple deduction of temperature rise by frictional heating on a translational reverse fault with planar fault plane. Such fault planes do exist in nature (e.g., figure 5.1 of Mukherjee 2014).

2. The model

Consider a block pushed up-dip along an inclined infinitely long plane of dip 'θ' ($\neq$0, 90°) with a constant velocity (v: relative velocity at which blocks move along fault plane with respect to each other) (figure 1). In this case of a dip-slip reverse fault, the hangingwall and the footwall blocks are assumed to be semi-infinite solids with friction induced constant flux of heat per unit time per unit area $\dot{Q}$ at the interface. As standard assumptions, the following considerations were made: (i) the work done by reverse fault movement is entirely converted to heat energy following the thermodynamic principle (Scholz 1990); and (ii) heat is transferred only perpendicular to the fault plane (Mase and Smith 1985). This assumption holds true when the hangingwall block slips and crosses the ramp and reaches the footwall flat. In this position, the isotherms are horizontal and heat transfers vertically. Like Graham and England (1976), faulting along a single plane is considered.

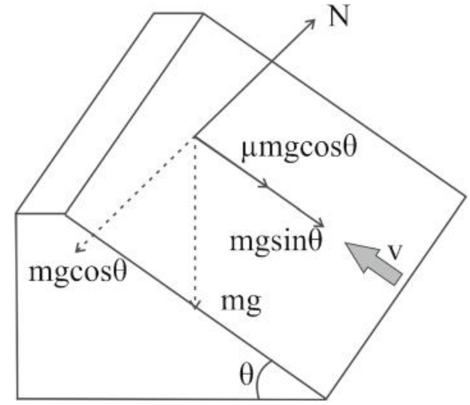

Figure 1. Footwall block of a dip-slip brittle reverse fault of dip 'θ'. When shear happens, the forces acting in different directions are resolved. μ: coefficient of friction; m: mass of the hangingwall block; g: acceleration due to gravity; N: normal force; v: shear velocity.

The stress due to frictional force developed at the interface between the two blocks (figure 1) is given as follows, as per Amonton's Law:

$$f = \mu N = \mu \rho g h \ \mathrm{Cos}\,\theta. \tag{1}$$

Here, N: normal stress; μ: coefficient of friction; ρ: density of the hangingwall block; g: acceleration due to gravity; and h: depth of the fault plane from the surface of Earth (thickness of the upper block). It can vary within the brittle regime of 0–15 km (Passchier and Trouw 2005). At a depth of more than 15 km, ductile deformation usually takes place, where equation (1) does not apply. To deduce shear heat/viscous dissipation in a ductile regime, see Mukherjee and Mulchrone (2013), Mulchrone and Mukherjee (2015, 2016), etc.

Rate of work done by frictional faulting per unit area: $\mu \rho g v h \ \mathrm{Cos}\,\theta$

where v is the relative velocity at which blocks move along fault plane with respect to each other.

Considering the standard assumption that the work done is completely converted into internal energy/heat, the rate of increase in internal energy per unit area in the system is given by the following equation:

$$\Delta \dot{E}_{\text{system}} = \mu \rho g v h \ \mathrm{Cos}\,\theta. \tag{2}$$

By the first law of thermodynamics, the rate of heat generated per unit area will be equal.

$$\dot{Q} = \mu \rho g v h \ \mathrm{Cos}\,\theta. \tag{3}$$

The equation of linear heat flow is

$$\dot{q} = -K\frac{\partial T}{\partial x},$$

(equation 4 of Jaupart and Mareschal 2011).

Here, K: thermal conductivity and T: temperature. The X-axis is perpendicular to the fault plane.

The equation of transient heat flow for a homogeneous solid whose conductivity does not depend on temperature can be written as the following:

$$k'\frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t}. \tag{4}$$

Heat flux also satisfies the same differential equation as T.

$$\frac{\partial \dot{q}}{\partial t} = k'\frac{\partial^2 \dot{q}}{\partial x^2} \quad \text{at } t > 0 \text{ and } x > 0. \tag{5}$$

Here, k': thermal diffusivity and $\dot{q} = \dot{Q}$ (constant) at x =0 and t>0.

Solving equation (5),

$$\dot{q} = \dot{Q}\,\mathrm{erfc}\frac{x}{2\sqrt{k't}},$$

(equation 6 of Jaupart and Mareschal 2011).

Here, t: duration of movement along fault plane, where error function, erf $x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-\xi^2}d\xi$.

$$\mathrm{ierfc}\,x = \int_x^\infty \mathrm{erfc}\,\xi \mathrm{d}\xi.$$

$$\mathrm{ierfc}\,x = \frac{1}{\sqrt{\pi}}e^{-x^2} - x\,\mathrm{erfc}\,x.$$

By integrating equation (4), for block A and taking boundary conditions as $T = 0$ at $t = 0$ and heat flux is equal to $\dot{Q}$ at $x = 0$.

Temperature rise by shearheating in hangingwall block,

$$T_1 = \frac{\dot{Q}_1}{K_1}\int_x^\infty \mathrm{erfc}\frac{x}{2\sqrt{k_1' t}}\mathrm{d}x \tag{6}$$

$$= 2\frac{\dot{Q}_1}{K_1}\sqrt{k_1' t}\,\mathrm{ierfc}\frac{x}{\sqrt[2]{k_1' t}} \tag{7}$$

$$= 2\frac{\dot{Q}_1}{K_1}\left\{\sqrt{\frac{k_1' t}{\pi}}e^{-\frac{x^2}{4k_1' t}} - \frac{x}{2}\mathrm{erfc}\frac{x}{2\sqrt{k_1' t}}\right\}. \tag{8}$$

Similarly for the footwall block,

$$T_2 = 2\frac{\dot{Q}_2}{K_2}\sqrt{k_2' t}\ \mathrm{ierfc}\frac{x}{\sqrt[2]{k_2' t}} \tag{9}$$

$$= 2\frac{\dot{Q}_2}{K_2}\left\{\sqrt{\frac{k_2' t}{\pi}}e^{-\frac{x^2}{4k_2' t}} - \frac{x}{2}\mathrm{erfc}\frac{x}{2\sqrt{k_2' t}}\right\}. \tag{10}$$

Here k_1': thermal diffusivity of hangingwall block and k_2': that of footwall block.

Let the distribution of heat generation per unit area in the two blocks due to faulting be $\dot{Q}_1$ and $\dot{Q}_2$. To deduct the heat distribution fractions $\dot{Q}_1$ and $\dot{Q}_2$, the boundary condition that temperature along the fault will be same for both the equations (8) and (10) is applied.

Putting $x = 0$ in both the equations and equating:

$$\frac{\dot{Q}_1\sqrt{k_1'}}{K_1} = \frac{\dot{Q}_2\sqrt{k_2'}}{K_2};$$

and

$$\dot{Q}_1 + \dot{Q}_2 = \dot{Q}.$$

By solving the above equations,

$$\dot{Q}_1 = \frac{\dot{Q}\,K_{1\sqrt{k_2'}}}{K_{2\sqrt{k_1'}} + K_1\sqrt{k_2'}}$$

and

$$\dot{Q}_2 = \frac{\dot{Q}\,K_{2\sqrt{k_1'}}}{K_{2\sqrt{k_1'}} + K_1\sqrt{k_2'}}.$$

By putting values of $\dot{Q}_1$ and $\dot{Q}_2$ in equations (7) and (9), respectively, one gets,

$$T_1 = \frac{2\,\dot{Q}\,K_{1\sqrt{k_1' k_2' t}}}{K_{2\sqrt{k_1'}} + K_1\sqrt{k_2'}}\ \mathrm{ierfc}\frac{x}{2\sqrt{k_1' t}}, \tag{11}$$

$$T_2 = \frac{2\,\dot{Q}\,K_{2\sqrt{k_1' k_2' t}}}{K_{2\sqrt{k_1'}} + K_1\sqrt{k_2'}}\ \mathrm{ierfc}\frac{x}{2\sqrt{k_2' t}}. \tag{12}$$

Putting $\dot{Q}$ from equation (3),

$$T_1 = 2\mu\rho g v h\ \mathrm{Cos}\,\theta K_1 k_1'^{0.5}k_2'^{0.5}t^{0.5}$$
$$\times \left(K_2 k_1'^{0.5} + K_1 k_2'^{0.5}\right)^{-1}$$
$$\times\,\mathrm{ierfc}\left\{0.5 \times k_1'^{-0.5}t'^{-0.5}\right\}, \tag{13}$$

$$T_2 = 2\mu\rho g v h\ \mathrm{Cos}\,\theta K_2 k_1'^{0.5}k_2'^{0.5}t^{0.5}$$
$$\times \left(K_2 k_1'^{0.5} + K_1 k_2'^{0.5}\right)^{-1}$$
$$\times\,\mathrm{ierfc}\left\{0.5 \times k_2'^{-0.5}t^{-0.5}\right\}. \tag{14}$$

Here, K_1 and K_2 are the thermal conductivities of the faulted hangingwall and footwall blocks.

3. Discussions and conclusions

Shear heating by brittle faults is relevant in different branches of geosciences. The temperature rise by shear heating for brittle dip-slip reverse fault is done here by considering a minimum number of physical parameters, viz., coefficient of friction (μ), density (ρ) and thickness (h) of hangingwall block, acceleration due to gravity (g), slip rate or slip velocity or shear velocity (v), thermal conductivities (K_i) and -diffusivities ($k_i^{'}$) of the two blocks, perpendicular distance from fault plane (x) and duration of slip (t). Faulting along a single sharp plane is considered here and not within a zone. Equations (13 and 14) show that the rise in temperature by shear heat (T_i) is proportional to the coefficient of friction (μ), the density (ρ) and thickness (h) of the hangingwall block. The first proportionality relation matches previous studies, e.g., Barr and Dahlen (1989). As rocks have various densities, shear heating would depend on the rock types. As the dip (θ) of the fault plane increases, T_i decreases. In other words, a low dipping thrust will have a high T_i. No such simple relation exists between T_i and thermal conductivity (K_i), nor with thermal diffusivity (k_i'). This is unlike viscous dissipation by ductile simple shear, where temperature rise is inversely proportional to the thermal conductivity (Mukherjee and Mulchrone 2013). Since the ierfc of an expression involving a parameter 'm', i.e., ierfc$\{m\}$ decreases drastically with increasing 'm', T_i keeps diminishing away from the fault plane and becomes insignificant after some distance. This matches with what has been stated by others, such as Wibberley *et al.* (2008).

An algebraic equation for temperature rise (T_i) by shear heat in brittle fault zones was presented by Hamada *et al.* (2009) as follows:

$$T_i = \tau\, v\, t\, w^{-1} C_P^{-1} \rho^{-1}. \qquad (15)$$

Here τ: shear stress; v: slip velocity; t: time; w: width of fault zone; C_P: specific heat at constant pressure; and ρ: density. Notice that unlike deduction for a sharp fault plane, T_i in a fault zone is simply proportional to the duration of slip. T_i was linked with the width of brittle shear/fault zones (Cardwell *et al.* 1978; Hamada *et al.* 2009; Fulton and Harris 2012). However, a brittle shear on a planar rough surface and not within a zone was considered. Therefore, the conclusions of Cardwell *et al.* (1978), Hamada *et al.* (2009) and Fulton and Harris (2012) cannot be cross-checked. Under a different physical condition, Lachenbruch (1986) pointed out that for a narrow shear zone, $T_i \propto t^{0.5}$ (also see Sibson 2002). However, this also does not match with the case, since the 't' term exists in equations (13 and 14) both inside and outside the 'ierfc function'.

For a planar fault with faulted blocks of the same lithologies adjacent to one another (i.e., $k_1' = k_2' = k'$; $K_1 = K_2 = K$ in equations (13 and 14)),

$$T_1 = T_2 = \mu\rho g h\ \mathrm{Cos}\,\theta\, v\, k'^{0.5} t^{0.5} \times \mathrm{ierfc}\,(0.5 \times k'^{-0.5} t^{-0.5}). \qquad (16)$$

Here, the rate of increase in heat per unit area of the two blocks becomes equal and is half the total amount (i.e., equal to $0.5\dot{Q}$), which is as expected. Several such algebraic expressions, such as of rate of heat produced per unit area (Sibson 2002; Turcotte and Schubert 2006), are available. The deduction (equation 13 and 14) and that by previous workers (e.g., equation 16) indicate that T_i will keep increasing as long as brittle shear continues. However, this does not happen in nature, since pore fluid modulates the temperature of the rocks (see Garagash and Rudnicki 2003a).

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