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# ON THE WAVELET ANALYSIS OF GEOMAGNETIC JERKS OF ALIBAG MAGNETIC OBSERVATORY DATA, INDIA

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#### ABSTRACT

Occurrences of Geomagnetic Jerks (GJs) in the decadal secular variation data have been understood to be random phenomena and show not only a varied spatio-temporal behaviour (sometimes occurring as a global feature and sometimes as a local one), but also a varied timefrequency localization (i.e., irregular occurrences of these high-frequency GJs in time). As a result, conventional signal processing techniques fail to facilitate a better understanding of the over all characteristics of GJs. Wavelet analysis is an effective mathematical tool to achieve the time-frequency localization of GJs. In the present study, we have applied continuous wavelet transformation to the uninterrupted time-series data of geomagnetic variations recorded at the Indian magnetic observatory, Alibag (ABG), over a period of 70 years (1930-2000). We have used different wavelet-families to address the following: (i) To identify optimal wavelet(s) that are suitable to characterize GJs and (ii) To check the presence of global jerks in ABG data that have been recorded at most of the worldwide observatories and also check the occurrences of local GJs (if any). Our results show that the 1968 global jerk could not be resolved well in ABG data by most wavelets, while the other global jerks that occurred in 1978 and 1992 were fairly well resolved. Among the variety of wavelets used, a set of 4 wavelets viz., Gaus3, Coif1, Coif2, and Sym4 could detect the global jerks and also some local jerks that occurred in 1943, 1951 and 1960. A set of three other wavelets, viz., Meyer, db8 and Morlet, could not detect any of the jerks, global or local.

*Key words*: Geomagnetic Jerks, Continuous Wavelet Transform, Alibag (India).

#### **1. INTRODUCTION**

Long period geomagnetic variations having periods of about one-year and above (except the 11-year solar cycle variations) owe their origin to the convection currents generated by the dynamo processes taking place within the Earth's liquid outer core. The first time-derivative of this field is known as geomagnetic secular variation. Examination of past records of geomagnetic data of several decades showed some sudden changes (or impulses) in the slope of the secular variations, having periods of about one year. Such features are known as Geomagnetic jerks (GJs). They are conspicuous in the East-West component of the magnetic field and are believed to intermittently occur sometimes globally (observed at most of the world magnetic observatories) and sometimes locally (observed at only a few observatories). The global jerks are reported to have occurred during the years, 1969-70, 1978-79 and 1991-92 and the local jerks during 1901-02, 1913-14, 1925-26, 1932-33, 1942-43, 1949-50 and 1999-2000. It is important to note here that the terms 'global' and 'local' are purely relative and it is not clear; how global is global? And how local is local? Such irregularly occurring and spatially varying geomagnetic phenomena have motivated a lot of interest in further understanding of their overall characteristics.

Origin of GJs has been the subject of a lot of debate. Employing Spherical Harmonic Analysis (SHA) technique, Malin and Hodder (1982) reported that the GJs are of internal origin. Later, claiming some loopholes in the SHA of Malin and Hodder, Alldredge (1984) argued that the GJs are of external origin. However, further studies by Gavoret et al. (1986). Nagao et al. (2002) and Bloxham et al (2002) showed the pronounced internal origin of GJs. Gavoret et al. (1986) separated the internal and external field components using geomagnetic indices and found a negligible influence of external sources on GJs. Nagao et al. (2002) applied statistical time series modeling to the monthly means of geomagnetic data and discussed the non-influence of external currents (Field Aligned Current and Ring Current) on GJs. Bloxham et al. (2002) explained that the GJs arise due to the combination of a steady flow and a simple time-varying. axisymmetric, equatorially symmetric, toroidal zonal flow of the core fluid. They also explained that the short duration of GJs is due to the differential fluid flow at the surface of the Earth's outer core, strongly signifying their internal origin. Studies of internal origin of GJs have led to understanding of their role on the lower mantle conductivity (Acache et al, 1980; Ducruix et al, 1980; Backus, 1983; Alexandrescu et al, 1999).

Most above studies required *a priori* assumption of the presence of GJs in the data. The GJs noted above clearly indicate that their occurrences are not periodic and that they occur both globally and locally. This implies that the conventional signal processing techniques fail to determine their time–frequency localization. Wavelet analysis on the other hand is a useful mathematical tool, which can determine the frequency of occurrence of

these geomagnetic impulses as a function of time. In other words, it helps to determine the time-frequency localization of GJs without any prior assumption of their presence in the signal. Exploiting this concept, Alexandrescu et al. (1995) first detected GJs in European magnetic observatory data. They designed their own wavelet, which is an approximate of the third derivative of Gaussian function. In the present study, we have done two independent exercises: First, we have attempted to determine optimum wavelet(s) that can best characterize global GJs. Second, we have examined the data of Indian magnetic observatory, Alibag (ABG), for the presence of world-wide reported jerks as well as local GJs (if any). In these two exercises, while the former requires an *a priori* assumption of occurrence of GJs in the analyzing signal, the latter does not. Detection of GJs depends on the degree of closeness of the shape of the analyzing wavelet (also known as mother wavelet) to that of the GJ itself. Different wavelets have different shapes and thus not all wavelets can identify GJs. We have used different wavelet-families to find out the optimal wavelet(s) that may best characterize the global and local geomagnetic jerks. The present study is first of its kind in Indian geomagnetic research. In the following sections, we sequentially describe the data and their initial processing, details of wavelet analysis, results and conclusions.

#### 2. DATA AND PROCESSING

Geomagnetic jerks are clearly evident in the East-West (EW) component of the magnetic field. Perhaps this is because, the influence of extra-terrestrial currents is relatively less on the EW component compared to that on the North-South (NS) and vertical components (McLeod, 1992). Hourly mean values of EW component of the magnetic field variations recorded during 1930-2000 at ABG (see Fig.1a for geographical location) have been obtained from WDC-C2 for Geomagnetism, Kyoto University, Japan. Data available in units of minutes were first converted to nT. From these, the daily means and then the monthly means were calculated by averaging the available number of days' data with equal number of days in that month. Monthly mean values have been used for the present study. At some places, wherever the whole month's data were missing, they were interpolated using the average of the previous and next month's values. Also occasional instrument noise and base line shifts (that usually result in the form of box-like jumps or step-like jumps) in the data were manually corrected prior to further analysis. However, such errors were found to be only a few in the whole length of data sequence. Thanks to the concerned staff at ABG for producing such a high quality data. Fully processed data are shown in Fig 1b.

#### **3. WAVELET ANALYSIS**

The wavelet function,  $\psi(t)$ , signifying the 'time-frequency' localization is defined by (Mallat, 1999)

$$\psi_{\tau,s}(t) = \frac{1}{\sqrt{s}} \psi\left(\frac{t-\tau}{s}\right).$$

where s > 0 indicates the scale and  $\tau$  indicates the translation parameter. Here, *s* is analogous to frequency, in the sense that higher scales (low frequencies) provide details of long-wavelength features of the signal and lower scales (high frequencies) provide details of the short-wavelength features of the signal. The translation parameter,  $\tau$ , refers to time information in the transformed domain. The function  $\psi(t)$  is called 'analyzing wavelet' or 'mother wavelet'. More details about the fundamentals of wavelet theory can be found in Daubechies (1992), Mallat (1989; 1999) and references therein.

In wavelet transformation, the signal to be transformed is multiplied with the mother wavelet and the transformation is computed for different segments of the data by varying  $\tau$  and s. The wavelet transformation in which,  $\tau$  and s will be continuously varied is called Continuous Wavelet Transformation (CWT) and the transformation in which, both  $\tau$  and s will be varied as power of an integer 'n' (i.e., n', j = 1, 2, 3, ..., k) is called Discrete Wavelet Transformation (DWT). Generally in DWT, dyadic scales and translations are used, in which case, n = 2. The distinction between CWT and DWT is that the former is linearly translational invariant. That means an amount of ' $\tau$ ' time shift in the wavelet function results in the same amount of shift in the transformed signal. As a result, transients can be effectively picked up by performing the computations at chosen variations in translations and scales. Whereas DWT is not linearly translational invariant. That means, in DWT, the translation in the input signal produces a translation in the output, only if the former is a multiple of the corresponding dyadic translation and scales of wavelet function<sup>1</sup>. So, if we apply DWT in the present study, there is a danger of skipping the whole signature of the jerk in such dyadic steps of translation and scale. Therefore, DWT is not suitable for the present study.

In analogy with the definition of inner product of two functions f(t) and g(t), given by

$$\langle f(t), g(t) \rangle = \int_{\mathbb{R}} f(t) \cdot g^*(t) dt$$

<sup>&</sup>lt;sup>1</sup>In such a case, that becomes 'dyadic wavelet transformation', which is always translational invariant.



Fig 1: (a) Geographical location of the Indian magnetic observatory, Alibag (ABG) and (b) The geomagnetic E-W component data (in nT) from 1930 to 2000.



Fig 2: Contour plot of CWT coefficients (scalograms) computed using the data shown in Fig. 1b for different wavelets. In the scalogram plots, the dark (light) colour represents low (high) coefficient values. The semi-vertical lines beneath each scalogram plot designate the associated lines of maxima (LoM) curves corresponding to the respective wavelet.

where  $f(t), g(t) \in L^2(R)$ . The CWT of the wavelet function  $\psi(t)$  and signal to be transformed, f(t), can be defined as

$$CWT(\tau,s) = \frac{1}{\sqrt{s}} \int f(t) \cdot \psi\left(\frac{t-\tau}{s}\right) dt \tag{1}$$

Equation (1) explains that the wavelet transformation gives a measure of the similarity between the signal and the wavelet function. Such a measure at any particular scale  $s_0$  and translation  $\tau_0$ , is identified by wavelet coefficient. The larger the value of this coefficient, the higher the similarity between the signal and the wavelet at ( $\tau_0$ ,  $s_0$ ) and vice-versa.

CWT was performed on the fully processed data using a variety of wavelets. Fig. 2 shows the contour plot of absolute CWT coefficients (called scalogram), corresponding to Gaus3, Coif1, Coif2 and Sym4 wavelets.

In the scalogram plots, the dark (light) colour represents low (high) coefficient values. In the plates beneath each scalogram, the semi-vertical lines designate Lines of Maxima (LoM). They are the curves formed by joining the points of local maxima at different scales. Meignen et al (2005) explained that the time localization of transients is better estimated with the LoM data than with the wavelet transform coefficients. These LoM signify discontinuities present in the data and that their origin at the lowest scale designates the time location of the occurrence of the discontinuity in the data. The paradox is, the geomagnetic jerks are represented by discontinuities, but not all discontinuities may represent jerks. Mallat and Hwang (1992) first described the wavelet transform modulus maxima method to detect the singularities in data. Alexandrescu et al (1995) by considering GJs as singularities and defining them as some  $\alpha^{\text{th}}$  derivative of the signal ( $\alpha$  being the regularity of the singularity), detected GJs from long period geomagnetic data by computing the coefficient of a liner regression between the logarithm of the absolute value of the CWT coefficient along the LoM and the logarithm of the scale, by modifying Mallat and Hwang's method. In the present study, we have followed Alexandrescu et al's approach. We give below a brief description of the same.

Since GJs manifest the second-time derivative of the geomagnetic field, they can be represented as some  $\alpha$ <sup>th</sup> derivative of the signal. So, the time-varying magnetic field, *f*(*t*) can be expressed as

$$f(t) = A(t + t_0)^{\alpha} + h(t) + n(t)$$
 (2)

Where h(t) is the long-period harmonic component, n(t) is the noise term as a function of time. The factor  $A(t+t_0)$  with the condition

$$(t+t_0)^{\alpha} = \begin{cases} 0 & t \le t_0 \\ (t+t_0)^{\alpha} & t > t_0 \end{cases}$$

defines the discontinuity in the signal centered around  $t_0$  having an amplitude A. Since the discontinuity is included in the signal (equation (2)), wavelet transformation can detect it and can also tell about the time of its occurrence. The above condition explains the *causal* nature of the occurrences of GJs, since each GJ is treated as an independent event.

Performing wavelet transformation on equation (2) and doing some subsequent simple mathematical calculations, we can obtain a linear regression between the logarithm of the absolute value of wavelet transform coefficients along the LoM and the logarithm of scale. Mathematically it is expressed in a simple form as

$$\log_2(|CWT(\tau,s)|) = \alpha \log_2(s) + \text{Constant} \quad (3)$$

where s > 0 is the dilation (scale) parameter. Equation (3) defines the regularity,  $\alpha$  of the geomagnetic jerk. Since the GJs manifest the second-time derivative of the geomagnetic field, the computed  $\alpha$  values should be close to 2. For full mathematical details of derivation of equation (3), the reader is referred to Alexandrescu et al (1995) and the references therein.

Equation (3) implies that the log-log plot of absolute value of the wavelet transform coefficient and the dilation should be a straight line with the slope  $\alpha$ . However, in practice, because of the noise present in the data, the curves slightly deviate from their straight line behaviour. We have evaluated equation (3) using individual LoM data of Fig. 2, corresponding to each wavelet and determined whether the LoM under study represented a GJ or not. For a LoM to designate a GJ, typically, the  $\alpha$  values should be close to 2. According to Mallat and Hwang (1992), the essential condition for a wavelet to detect a singularity in a given data set is that the number of its vanishing moments should be greater than  $\alpha$ . A brief description of all the wavelets used in the present study and their properties are given in Appendix -A.

Results of the present study show that among the variety of wavelets used, a set of four wavelets, viz., Gaus3, Coif1, Coif2 and Sym4 showed a close linear relationship between the logarithms of the absolute value of the wavelet transform coefficient and the scale corresponding to 1968, 1978 and 1992 jerk events (Fig. 3). These four wavelets have also detected a few local jerks in ABG data that occurred in 1943, 1951 and 1960.



**Fig 3**: Plots depicting the linear relationship between the logarithm of the absolute value of the CWT coefficient and the logarithm of the scale corresponding to the global jerks of 1968, 1978 and 1992 computed using the respective LoM data of each wavelet using equation (3). The linearity in the curves appears to be fairly good beyond the dilation values ranging between 2<sup>4</sup> and 2<sup>5</sup>. Below this dilation range, the data are believed to have been affected by external noise. Note the poor resolution of the 1968 global jerk by Coif1, Coif2 and Sym4 wavelets. This is a clear manifestation of the fact that a wavelet that can detect a discontinuity in a given data, may not be able to detect others (if present) in the same data. See Appendix-A for shapes of these wavelets and their properties.



Fig 4: Same as Fig. 3, but, for the local jerks that occurred in 1943, 1951 and 1960.



Fig 5: (a) Scalograms and lines of maxima plots for ABG data determined for the three wavelets, viz., Meyer, Db8 and Morlet, which did not detect any of the jerks, global or local. (b) Curves generated using equation (3) for all the global and local jerks using the above three wavelets. See Appendix-A for shapes of these wavelets and their properties.

The nature of these jerks together with their estimated  $\alpha$  values is shown in Fig. 4. We have found another set of three wavelets, which could not detect any of the jerks, global or local. They are Meyer, db8 and Morlet wavelets. The scalograms and the respective LoM of each of these wavelets are shown in Fig. 5a and the behaviour of these jerks in accordance with equation (3) is shown in Fig. 5b. The  $\alpha$  values shown in each plate in Figs. 3, 4, and 5b are calculated by fitting a straight line to the linear portion of the respective curves.

#### 4. RESULTS AND DISCUSSION

We discuss below two main issues concerning our study, viz., (i). The choice of mother wavelet and (ii) the detection of global and local jerks in ABG data.

In the first place, one can question the crude method of applying many wavelets to the data to determine the optimum ones that can best characterize the jerks. Isn't there any refined procedure to achieve the desired objective? The answer is a straightforward *no*.

Although, there are a number of wavelets that have different properties and yet can satisfy the essential mathematical criteria (admissibility<sup>2</sup> and regularity<sup>3</sup> conditions) for them to be defined as wavelets, there are only a few, which can detect singularities in a given data set. In other words, not all wavelets can detect GJs. Further, even among those few that can detect singularities, it is possible that a wavelet, which can detect a singularity in the given data, may fail to detect another one (if exists) in the same data (Fig. 3). Secondly, by observing the shape of the wavelet and its properties, it is difficult to ascertain whether a chosen wavelet will be useful for the intended study or not. For example, the Morlet wavelet, which is found to be very effective in delineating the depths to the top of the hydrocarbon zones when applied to geophysical well-log data (Choudhury et al., 2007) is found to be not suitable to identify the jerks in the present study (Fig. 5). Therefore, we suggest that unless one is designing one's own wavelet, it is advised to test all the available wavelets to choose an optimum wavelet suitable for one's analysis. Such a formidable exercise is unavoidable, if we have to obtain a good time-frequency resolution for the singularity under investigation. Although various workers have discussed several methods of choosing optimum wavelets to analyze signals of their interest (see for e.g. Ahuja et al., 1995; Kumar and Foufoula-Georgiou, 1997; Nenadic and Burdick, 2005), their applicability needs to be tested on individual data sets and thus an apt choice of wavelets could be made only on case-by-case basis. In other words, the choice of wavelet is dictated by the objectives of the study.

In the LoM plots of Fig. 2, the origin of the LoM at the lowest scale designates the times of occurrences of the discontinuities in the data. It is interesting to observe that the LoM corresponding to the identified global and local jerks are distinctly separate and that they do not overlap with the adjacent ones except at the ends, which is due to edge effects of the signal.

At a first glance, one can observe from Fig. 3 that, of the three global jerks occurred in 1968, 1978 and 1992, only the latter two could be resolved fairly well by a set of four wavelets, Gaus3, Coif1, Coif2 and Sym4, while the former is resolved only by the Gaus3 wavelet. This is a clear manifestation of the fact that a wavelet that can detect a singularity in a given data, may fail to detect others (if present) in the same data. The ability for a wavelet to detect a singularity in a given data depends on the degree of closeness of the shape of the wavelet to that of the singularity under investigation. The fact that a wavelet could detect a singularity in a given data and fails to detect another in the same data implies that the shapes of the different singularities present in the data are different. This poses a question: Do the naturally occurring geomagnetic jerks possess different shapes at different times of their occurrences? We attempt to answer this question in our further study on GJs. Further, as shown in Fig. 4, the above set of four wavelets could well resolve the local jerks occurred in 1943, 1951 and 1960 and that their  $\alpha$  values, on the whole, corresponding to these local jerks have been consistent.

The curves shown in Figs. 3 and 4 tend to be linear (except those of the 1968 global jerk in Fig. 3 corresponding to Coif1, Coif2 and Sym4 wavelets) beyond the dilation (S) value of 16-32 (i.e.,  $2^4$  to  $2^5$ ) and below this value the curves are not linear. This marks the transition between the part of the curve dominated by external noise (corresponding to  $s < 2^4$ ) and the part dominated by the GJ ( $s > 2^4$ ). If the external noise is dominant, then the transition value on the scale axis will be high and the resultant regularity will be less accurate. However, for a couple of curves in Fig. 4 these transition values on the dilation axis are less than  $2^4$ . This may possibly suggest that the wavelet may be averaging out the external noise during the CWT process. Further, in Figs. 3 and 4 the linear nature of the curves at higher dilations is less perfect, and thus some kinks are observed in the linear portion of the curves in the dilation range  $2^4$ to  $2^5$ . We are not sure, if that could be due to the presence of strong long-period external noise in the data. Further, it is also important to note that the  $\alpha$  values estimated

<sup>&</sup>lt;sup>2</sup>Admissibility condition implies that the wavelet must be oscillatory and that its average value in time domain must be zero.

<sup>&</sup>lt;sup>3</sup>Regularity condition explains that the wavelet must be localized in time and thus it must be finite in length.

from equation (3) are not the intrinsic property of the jerk itself, but also depend on the wavelet used.

The curves corresponding to the LoM originating at the years 1943, 1951 and 1960 clearly show a well-defined log-linear nature (Fig. 4). Since global jerks have not been reported to have occurred in these years, we believe that these may be local jerks. In 1949, a local jerk has been reported to have occurred in Pacific and American regions (Michelis and Tozzi, 2005). Based on the results of the present study, we are unable to ascertain, weather the 1951 jerk seen in ABG data, which occurred after a time lag of two years, is a continuation of the 1949 jerk seen elsewhere, or is a completely independent one. If the former were to be true, then such late appearances of jerks have earlier been reported to be due to mantle conductivity filtering effect (Backus, 1983). However, further studies are currently in progress to have a correct understanding of the nature of these local jerks.

Interestingly, we have found another set of three wavelets, viz., Meyer, db8 and Morlet, which did not detect any of the global or local jerks. The plots generated using equation (3) corresponding to these wavelets for the above reported global and local jerks are shown in Fig. 5b. At present we only believe that the reason for failure of these wavelets to detect the jerks could be due to the poor coherency between the shapes of these wavelets and that of the global and local jerks. Further studies are currently being pursued to quantify such observations.

#### 5. CONCLUSIONS

Wavelet analysis is a powerful tool to identify discontinuities like geomagnetic jerks in the decadal variations of geomagnetic data. It is very effective in determining the spatial and temporal characteristics of geomagnetic jerks. Its uniqueness also lies in its ability to time-frequency localization determine the of geomagnetic jerks without any prior assumption of their presence in the signal. However, the difficult task in wavelet analysis is to determine the choice of mother wavelet. Most studies adopt the trial and error method to decide on the choice of wavelet (Ahuja et al., 1995). Our investigations confirm that the choice of the wavelet is governed by the objectives of study. Also, for the first time, we have identified the presence of local jerks in ABG data that occurred during the years 1943, 1951 and 1960 and these are the first results on geomagnetic jerk phenomena using Indian data. Among the limited number of wavelets used for the present study, we found that Gaus3, Coif1, Coif2 and Sym4 wavelets could fairly resolve the presence of global jerks (occurred in 1968, 1978 and 1992). The same set of wavelets could also well resolve the local jerks present in the ABG data. We also

have found a set of three other wavelets, viz., Meyer, db8 and Morlet, which could not detect any of the jerks, global or local. Further investigations are currently in progress to address some of the important issues concerning geomagnetic jerk phenomena discussed above and understand them in a broader perspective. In the envisaged study, we propose to use more wavelets and use global and other Indian magnetic observatory data sets to characterize and help improve the understanding of the GJ Phenomena both on global and local scales. A long-term goal of these ongoing studies is to estimate the constraints on the lower mantle conductivity below the Indian plate through the analysis of geomagnetic jerks.

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#### APPENDIX – A

Basic definitions of some properties of the wavelets used in the present study – A quick reckoner. For more mathematical details of these and other wavelets, the reader is referred to Mallat (1999), Kumar and Foufoula Georgiou (1994) to cite a few

Symmetry: The symmetry property of wavelets explains that the wavelet transform (WT) of the mirror (m) of a signal is mirror of the wavelet transform of the signal. i.e.,  $WT\{m[f(t)]\} = m\{WT[f(t)]\}$ .

*Compact support*: This property explains that the wavelet vanishes outside a finite interval. The shorter the interval is, the compact the wavelet is.

*Orthogonality*: This property implies that if it is possible to construct some wavelets  $\psi(t)$ , such that  $\psi_{\mu\nu}(t)$  are

orthonormal, i.e.,  $\int \psi_{u,v}(t) \psi_{u',v'}(t) dt = \delta_{uu'} \delta_{vv'}$ ,

where  $\delta_{i,j}$  is the delta function defined as  $\delta_{i,j} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{otherwise} \end{cases}$ then, this implies that such wavelets are orthogonal to their "*translates*" and "*dilates*".

*Vanishing Moments*: A wavelet,  $\psi(t)$  has "N" vanishing moments, if the Fourier transform of the wavelet at the origin, is *k* times continuously differentiable. i.e.,  $\frac{d^k}{d^k}\psi(\omega = 0) = 0 \text{ for } k = 0.1 \text{ N-1}$ 

$$\frac{d}{d\omega^k}\psi(\omega=0) = 0$$
, for *k*=0, 1,..., N-1.

Name of the Wavelet	Shape of the Wavelet *	Symmetry	Compact Support	Orthogonality	Vanishing Moments
Gaussian 3		Anti-Symmetric	No	No	Arbitrary
Coiflet 1		Near Symmetric	Yes	Yes	2
Coiflet 2		Near Symmetric	Yes	Yes	4
Symlet 4		Near Symmetric	Yes	Yes	4
Meyer		Symmetric	No	Yes	Arbitrary
Daubechies 8		No	Yes	Yes	8
Morlet		Symmetric	No	No	Arbitrary

\* The horizontal and vertical axes for the wavelet shapes are in arbitrary units.